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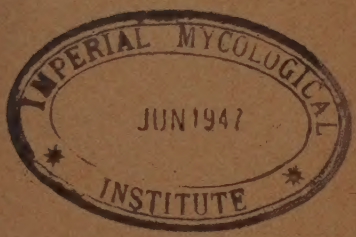

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MATHÉMATIQUES

SUR UN THÉORÈME DE F. NOETHER

Par S. MICHLIN

(Présenté par S. L. Soboleff, de l'Académie, le 28.IX.1943)

Dans un mémoire (1) de F. Noether on trouve, entre autres, le théorème suivant:

Pour que l'équation intégrale singulière

$$a(x)\varphi(x) - \frac{b(x)}{2\pi}\int_{-\pi}^{\pi}\varphi(s)\operatorname{ctg}\frac{s-x}{2}\,ds + \int_{-\pi}^{\pi}K(x,s)\varphi(s)\,ds = F(x) \qquad (1)$$

possède une solution, il faut et il suffit que son second membre soit orthogonal à toutes les solutions de l'équation homogène conjuguée.

On suppose que le noyau $K(x, s)$ est régulier, $a(x)$ et $b(x)$ vérifient la condition de Lipschitz, et toutes les fonctions qui figurent dans l'équation sont réelles. Cette dernière hypothèse est essentielle pour le raisonnement de F. Noether. W. Koupradze (2)* en conservant les mêmes hypothèses a donné une démonstration un peu différente pour le théorème de F. Noether. Dans la présente Note nous donnons une démonstration plus simple pour le même théorème et nous admettons aussi les fonctions complexes.

Pour abréger nous écrirons

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\varphi(s)\operatorname{ctg}\frac{s-x}{2}\,ds = p\varphi \qquad (2)$$

L'opérateur $p\varphi$ est linéaire dans $L_2(-\pi, \pi)$; sa norme est égale à l'unité. Nous allons considérer maintenant l'équation

$$A\varphi = a(x)\varphi(x) - b(x)\,p\varphi + t\varphi = F(x) \qquad (3)$$

On désigne par $t\varphi$ un opérateur arbitraire totalement continu dans $L_2(-\pi, \pi)$. En ce qui concerne $a(x)$ et $b(x)$ nous supposerons que ces fonctions vérifient la condition de Lipschitz, et que $a^2(x) + b^2(x)$ ne s'annule pas sur le segment $(-\pi, \pi)$.

La nécessité de la condition qui se trouve dans le théorème de F. Noether est un fait très général: si une équation $A\varphi = F$ (A est un opérateur linéaire arbitraire dans l'espace de Gilbert) possède une solution, et ψ est une solution de l'équation homogène conjuguée $A^*\psi = 0$ ** on a

$$(F, \psi) = (A\varphi, \psi) = (\varphi, A^*\psi) = 0$$

Démontrons la suffisance. Un rôle essentiel appartient au lemme suivant.

* L'article (2) contient quelques erreurs non essentielles (l'opérateur L doit être remplacé par l'opérateur $\overline{K}$). C'est W. Koupradze lui-même qui m'a indiqué la nécessité des corrections.

** Ici et dans la suite on désigne toujours par un astérisque l'opérateur conjugué.

Lemme. *Soit*

$$Q\varphi = \varphi(x) - \mu(x)\, p\varphi + t\varphi \tag{4}$$

où $t\varphi$ *est un opérateur totalement continu, et* $\mu(x)$ *vérifie la condition de Lipschitz et son module est strictement inférieur à* 1. *Alors*

1) *pour l'équation*

$$Q\varphi = F(x) \tag{5}$$

l'alternative de Fredholm a lieu;

2) *les équations* $Q\varphi = 0$ *et* $Q^*\varphi = 0$ *ont un même nombre de solutions linéairement indépendantes;*

3) *pour que l'équation* (5) *soit résoluble, il suffit que* $F(x)$ *soit orthogonale à toutes les solutions de l'équation* $Q^*\varphi = 0$.

La démonstration de ce lemme est facile à obtenir, si l'on remarque qu'il existe un opérateur T inverse à l'opérateur

$$\varphi(x) - \mu(x)\, p\varphi$$

et que l'équation $TQ\varphi = TF$ est une équation du type de Fredholm équivalente à (5).

Introduisons encore dans nos considérations l'opérateur A^* conjugué à A

$$A^*\psi = \overline{a(x)}\,\psi(x) + p(\overline{b}, \psi) + t^*\psi$$

Il peut être présenté dans la forme

$$A^*\psi = \overline{a(x)}\,\psi(x) + \overline{b(x)}\, p\psi + t_1\psi \tag{6}$$

où

$$t_1\psi = t^*\psi + [p(\overline{b\psi}) - \overline{b(x)}\, p\psi]$$

est un opérateur totalement continu. En appliquant l'opérateur A^* aux deux membres de l'équation (3), nous aurons une nouvelle équation qui peut être écrite dans la forme

$$\frac{A^*A\varphi}{|a^2(x)| + |b^2(x)|} = \varphi(x) - \mu(x)\, p\varphi + t_2\varphi = \frac{A^*F}{|a^2(x)| + |b^2(x)|} \tag{7}$$

où $t_2\varphi$ est un opérateur totalement continu, et

$$\mu(x) = -\frac{2J(\overline{a}(x)\, b(x))}{|a^2(x)| + |b^2(x)|} \tag{8}$$

La quantité $|\mu(x)|$ est inférieure à 1; elle n'est égale à 1 que dans le cas où $a = \pm bi$, et nous avons supposé que ce cas est exclu.

Démontrons que l'équation (7) est résoluble. A cet effet, en vertu du lemme, il suffit de démontrer que son second membre est orthogonal à toutes les solutions de l'équation homogène conjuguée. Il est facile à démontrer que cette condition est équivalente à la suivante: la fonction A^*F est orthogonale à chaque fonction ψ vérifiant l'équation $A^*A\psi = 0$. Mais les solutions de cette équation coïncident avec les solutions de l'équation $A\psi = 0$. En effet, si $A^*A\psi = 0$, on a

$$0 = (A^*A\psi, \psi) = (A\psi, A\psi) \tag{9}$$

d'où $A\psi = 0$. Mais si $A\psi = 0$, on a

$$(A^*F, \psi) = (F, A\psi) = 0$$

et il est ainsi démontré que l'équation (7) est résoluble.

Supposons maintenant que la condition du théorème de Noether est vérifiée de sorte que l'on a

$$(F, \omega) = 0 \quad \text{si} \quad A^*\omega = 0 \tag{10}$$

Démontrons que dans ce cas l'équation (3) est résoluble, à savoir, que toutes les solutions de l'équation (7) sont aussi des solutions de (3). Présentons l'équation (7) dans la forme

$$A^*(A\varphi - F) = 0$$

Soit φ une solution de cette équation. Posons $A\varphi - F = \omega$, de sorte que $A^*\omega = 0$. Nous aurons

$$\| \omega \|^2 = (\omega, \omega) = (A\varphi - F, \omega) = (\varphi, A^*\omega) - (F, \omega) = 0$$

d'où $\omega = 0$.

Le théorème est démontré.

Remarque. Il suit de nos raisonnements que si la condition (10) est vérifiée, les équations (3) et (7) sont équivalentes. Ce fait est intimement lié au théorème suivant:

Si $A\varphi$ est un opérateur linéaire dans l'espace de Gilbert et si l'équation $A\varphi = F$ est résoluble, elle est équivalente à l'équation $A^*A\varphi = A^*F$.

Soit φ_1 un vecteur qui vérifie l'équation $A\varphi = F$. Il vérifie aussi l'équation $A^*A\varphi = A^*F$. Supposons que le vecteur φ_2 vérifie la même équation. Démontrons que φ_2 vérifie aussi l'équation $A\varphi = F$.

Posons $\varphi_2 - \varphi_1 = \omega$. Alors $A^*A\omega = 0$ et, en vertu de la formule (9), on a $A\omega = 0$. Mais alors $A\varphi_2 = A\varphi_1 = F$, et le théorème est démontré.

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LITTÉRATURE CITÉE

[1] F. Noether, Math. Ann., 82 (1921). [2] W. Koupradze, C. R. Acad. Sci. Géorgienne, II, No. 1—2 (1941).

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MATHÉMATIQUES

SUR UN NOUVEAU TYPE DE TABLEAUX DE FONCTIONS À PLUSIEURS VARIABLES

Par L. NEUSCHULER

(Présenté par S. L. Soboleff, de l'Académie, le 20.XI.1943)

Les tableaux ordinaires pour les fonctions à plusieurs variables, où le nombre d'entrées est égal au nombre de variables, sont assez volumineux, même si le nombre nécessaire des valeurs des arguments est petit, et à partir d'un certain nombre de ces valeurs les volumes de ces tableaux dépassent les limites de ce qui est pratiquement réalisable.

La construction des tableaux non pas pour toutes les valeurs nécessaires des arguments, mais pour une certaine suite présente tous les inconvénients liés à l'interpolation par rapport à plusieurs variables.

Dans la présente Note nous indiquons des schèmes de constructions des tableaux pour les fonctions à plusieurs variables, schèmes qui permettent souvent de réaliser des tableaux de volume convenable (sans qu'il soit nécessaire d'interpoler), et pour un tel nombre de systèmes des valeurs des arguments qu'il serait impossible de considérer pratiquement dans le cas de constructions ordinaires. D'ailleurs, dans ces constructions le procédé pour opérer (qui dans le cas des tableaux n'exigeant pas d'opérations arithmétiques est déterminé par le nombre d'entrées du tableau) reste presque le même que dans le cas des tableaux ordinaires.

Supposons qu'une fonction à quatre variables $f(x_1, x_2, x_3, x_4)$ admet une représentation (exacte ou bien approchée autant que l'on veut)

$$f(x_1, x_2, x_3, x_4) = F(u, v) \tag{1}$$

où

$$u = f_1(x_i, x_j), \quad \text{et} \quad v = f_2(x_p, x_q) \quad (i, j, p, q = 1, 2, 3, 4)$$

et supposons que si l'on détermine les valeurs de f avec la précision exigée, on a:

1) $u = f_1(x_i, x_j)$ prend la suite des valeurs $a_1, a_2, \ldots, a_{m_1}$;
2) $v = f_2(x_p, x_q)$ prend la suite des valeurs $b_1, b_2, \ldots, b_{m_2}$;
3) x_i prend la suite des valeurs $c_1, c_2, \ldots, c_{m_3}$;
4) x_p prend la suite des valeurs $d_1, d_2, \ldots, d_{m_4}$.

Construisons un tableau de la forme (A) (p. 143).

Dans ce tableau:

1) dans le premier membre dans la ligne m et la colonne n on place la valeur de x_j vérifiant l'égalité

$$a_m = f_1(c_n, x_j)$$

2) dans le second membre dans la ligne s, la colonne t et la colonnette x_q on place la valeur de x_q vérifiant l'égalité

$$b_t = f_2(d_s, x_q)$$

3) dans le second membre dans la ligne α, la colonne β et la colonnette on place la valeur

$$F(\alpha, \beta)$$

Il suit de ce qui précède que le tableau de la forme (A) permet de trouver les valeurs de $f(x_1, x_2, x_3, x_4)$.

En effet, si $x_i = c_{k_1}$ et $x_p = d_{k_2}$, on peut trouver la valeur de $f(x_1, x_2, x_3, x_4)$ du tableau (A) de la manière suivante:

1) dans le premier membre du tableau (A) dans la colonne c_{k_1} on trouve dans une certaine ligne γ la valeur x_j;

2) dans le second membre du tableau dans la ligne d_{k_2} on trouve dans une certaine colonne δ la valeur x_q;

3) dans le second membre du tableau dans la colonnette f sur l'intersection de la ligne γ et de la colonne δ on trouve la valeur cherchée $f(x_1, x_2, x_3, x_4)f$

Premier membre* Second membre*

u \ x_i	$c_1 c_2 c_3 \ldots c_{m_3}$	x_p \ v	b_1	b_2	$\ldots$	b_{m_2}
			$x_q f$	$x_q f$	$\ldots$	$x_q f$
a_1		d_1				
a_2		d_2				
$\cdot$		$\cdot$				
$\cdot$		$\cdot$				
a_{m_1}		d_{m_4}				

(A)

* Les valeurs de la fonction u qui désignent les lignes dans le premier membre, et les valeurs de v qui désignent les colonnes du second membre du tableau (A) ne sont pas nécessaires pour déterminer les valeurs de f. On les donne pour expliquer la construction. Il y a ici une analogie avec les échelles muettes dans la monographie.

Si les nombres m_1, m_2, m_3, m_4 (appelés dans la suite paramètres de volume du tableau) sont tels que le tableau de la forme (A) peut être placé sur une page déployée dont le format est admissible pour l'usage, alors cette forme du tableau donne évidemment la solution de la question, quelle est la disposition la plus économique des valeurs nécessaires de la fonction.

L'économie dans le volume du tableau (A) en comparaison avec le tableau ordinaire a pour cause essentielle le fait que le nombre des valeurs différentes de précision donnée (désignons les par m_1 et m_2) des fonctions

$$u = f_1(x_i, x_j) \text{ et } \varphi = f_2(x_p, x_q)$$

est souvent essentiellement inférieur respectivement au nombres N_1 et N_2 des systèmes de valeurs des arguments x_i et x_j, et x_p et x_q.

Le procédé pour opérer avec le tableau (A) se réduit réellement aussi à quatre entrées comme dans le tableau ordinaire, mais il y a cette différence que le tableau ordinaire (dans le cas où le format serait le même) devrait avoir trois dimensions, donc il exigerait un feuillettement.

Si les rapports m_1/N_1 et m_2/N_2 sont assez petits, la construction du tableau de la forme (A) a aussi un sens dans le cas, où quelques-uns des paramètres de volume du tableau m_1, m_2, m_3, m_4 ou tous ces paramètres sont tels qu'il est impossible de les disposer sur un format donné.

Dans ce cas on opère de la manière suivante:

1) on décompose en parties les intervalles de la variation des arguments x_i et x_j de la fonction $u = f_1(x_i, x_j)$ et des arguments x_p et x_q de la fonction $\varphi = f_2(x_p, x_q)$;

2) on construit des tableaux de la forme (A) pour toutes les réunions possibles de ces intervalles partiels.

Le choix de ces intervalles est assujetti à cette condition que le volume total de l'ensemble des tableaux de la forme (A) pour tous les intervalles nécessaires soit minimal quand le format est donné.

Si le volume que l'on obtient de cette manière est essentiellement inférieur au volume du tableau ordinaire, la construction d'un tel tableau est déjà justifiée, car le procédé pour opérer avec un tel tableau est à peu près le même qu'avec le tableau ordinaire.

Comme le plus simple exemple d'une fonction à quatre variables dont on peut disposer les valeurs en un tableau de la forme (A), on peut indiquer la fonction linéaire

$$u = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4.$$

Si la fonction de 4 variables $u = f(x_1, x_2, x_3, x_4)$ n'admet pas de représentation (1), mais elle admet la représentation de la forme

$$u = f(x_1, x_2, x_3, x_4) = \Phi\{F[f_1(x_i, x_j),\ x_p],\ x_q\} \qquad (2)$$

on peut construire un tableau de la forme (A) pour la fonction

$$x_p = F_1[f_1(x_i, x_j),\ \varphi_1(u, x_q)] \qquad (3)$$

où (3) est équivalent à (2), et utiliser ce tableau pour trouver les valeur $u = f$ de la manière suivante:

1) dans le premier membre du tableau dans la colonne x_i on trouve dans une certaine ligne α la valeur x_j;

2) dans la ligne α du second membre du tableau on trouve dans une certaine colonne β la valeur x_p;

3) dans cette colonne β dans l'intersection avec la ligne x_q on trouvera évidemment la valeur de u cherchée.

Le principe de la construction des tableaux de la forme (A) pour les fonctions à 4 variables admettant la représentation (1), s'étend facilement aux fonctions à 6 variables $u = f(x_1, x_2, x_3, x_4, x_5, x_6)$ admettant la représentation

$$u = f(x_1, x_2, x_3, x_4, x_5, x_6) = \Phi\{F[f_1(x_i, x_j),\ f_2(x_p, x_q)],\ f_3(x_s, x_r)\} \qquad (4)$$

$$u = f(x_1, x_2, x_3, x_4, x_5, x_6) = \Phi(F\{\varphi[f_1(x_i, x_j),\ f_2(x_p, x_q)],\ x_s\},\ x_r) \qquad (5)$$

ou bien

$$u = f(x_1, x_2, x_3, x_4, x_5, x_6) = \psi(\varphi(\Phi\{F[f_1(x_i, x_j),\ x_p],\ x_q\},\ x^s),\ x_r) \qquad (6)$$

Considérons d'abord les fonctions de la forme (4).

Supposons que pour la précision exigée des valeurs de f

1)	$u = f_1(x_i, x_j)$	prend	la	suite	des	valeurs	$A_1, A_2, \ldots, A_{m_1}$
2)	$v = f_2(x_p, x_q)$	»	»	»	»	»	$B_1, B_2, \ldots, B_{m_2}$
3)	$w = f_3(x_s, x_r)$	»	»	»	»	»	$C_1, C_2, \ldots, C_{m_3}$
4)	$x_i =$	»	»	»	»	»	$a_1, a_2, \ldots, a_{n_1}$
5)	$x_p =$	»	»	»	»	»	$b_1, b_2, \ldots, b_{n_2}$
6)	$x_s =$	»	»	»	»	»	$c_1, c_2, \ldots, c_{n_3}$

Construisons un tableau formé d'une page (ou bien d'une page déployée) de la forme (B) [qui ne diffère de la forme (A) que par l'absence de la colonnette f]:

Premier membre		Second membre	
u \ i	$a_1\ a_2 \dots a_{n_1}$	x_p \ v	$B_1\ B_2 \dots B_{m_2}$ / $x_q\ x_q \dots x_q$
A_1		b_1	
A_2		b_2	
⋮		⋮	
A_{m_1}		b_{n_2}	

(B)

et un tableau formé de m_3 pages (ou bien de pages déployées) dont la N-ième est de la forme (C):

$x_r =$	
$x_s =$	$c_1\ c_2 \dots c_{n_3}$
	$1\ \ 2 \dots m_3$
1	
2	
⋮	
m_1	

(C)

où dans la ligne des x_s on a au-dessus du nombre $x_s = c_k$ la valeur x_r qui vérifie avec la précision exigée l'égalité

$$C_N = f_3(c_k, x_r)$$

et où dans l'intersection de la ligne k_1 et de la colonne k_2 on place les valeurs

$$\Phi[F(A_{k_1}, B_{k_2}),\ C_N]$$

Le procédé pour opérer avec les tableaux (B) et (C) pour déterminer les valeurs de $u = f(x_1, x_2, x_3, x_4, x_5, x_6)$ est, ainsi qu'il suit de leur construction, le suivant:

1) dans le premier membre du tableau (B) dans la colonne marquée par la valeur donnée de x_i, on trouve dans une certaine ligne m la valeur nécessaire de x_j;

2) dans le même tableau (B) dans le second membre dans la ligne marquée par la valeur donnée x_p et dans une certaine colonne n on trouve le x_q nécessaire;

3) sur une certaine page N du tableau (C) on trouve le x_r nécessaire parmi les nombres de la ligne x_r marquée par la valeur x_s donnée.

Sur la même page N sur l'intersection de la ligne m et de la colonne n on trouve le $u = f(x_1, x_2, x_3, x_4, x_5, x_6)$.

Si la fonction de 6 variables $u = f(x_1, x_2, x_3, x_4, x_5, x_6)$ n'admet pas de représentation de la forme (4), mais admet une représentation de la forme (5), on peut agir d'une manière analogue à celle qui a servi pour former les tableaux des fonctions de la forme (2).

Supposons qu'en résolvant (5) par rapport à x_s on trouve:

$$x_s = \Phi_1\{\varphi[f_1(x_i, x_j),\ f_2(x_p, x_q)],\ \varphi(u, x_r)\} \tag{7}$$

Construisons pour (7) un tableau de la forme (B) et un tableau (C′) qui diffère du tableau (C) pour une fonction de la forme (4) en ce que les valeurs x_s sont remplacées par les valeurs x_r, et les valeurs x_r par les valeurs respectives de u. La manière d'utiliser ces tableaux pour déterminer les valeurs

de la fonction $u = f(x_1, x_2, x_3, x_4, x_5, x_6)$, admettant une représentation de la forme (5), est la suivante:

1) dans le premier membre du tableau (B) dans la colonne x_l et une certaine ligne m nous trouvons le x_j nécessaire, et dans le second membre du tableau (B) dans la ligne x_p dans une certaine colonne n nous trouvons le x_q nécessaire;

2) dans le tableau (C') sur une certaine page N on troure le x_s nécessaire * sur l'intersection de la ligne m et de la colonne n;

3) sur la même page N dans la ligne marquée par u au-dessus de la valeur x_r donnée on trouve le u cherché.

La construction des tableaux de la structure indiquée pour les fonctions de 6 variables admettant une représentation de la forme (6), est aussi possible, comme il est facile à voir, par une combinaison des méthodes appliquées pour former les tableaux des fonctions de la forme (2) et des fonctions de la forme (5).

Les constructions des tableaux pour les fonctions à 4 variables que nous avons considérées ici peuvent aussi être appliquées pour former les tableaux des fonctions à 3 variables.

Ainsi si une fonction à 3 variables admet une représentation (précise ou approchée avec une précision suffisante) de la forme

$$f(x_1, x_2, x_3) = F[f_1(x_i, x_j), f_2(x_j, x_p)] \tag{8}$$

ou bien

$$f(x_1, x_2, x_3) = \Phi\{F[f_1(x_i, x_j), x_p], x_i\}^{**} \tag{9}$$

on peut construire pour cette fonction des tableaux de la même structure que ceux qui ont été construits pour les fonctions à 4 variables admettant respectivement des représentations de la forme (1) ou (2).

En concluant, remarquons que les constructions des tableaux considérées ici peuvent être représentées graphiquement.

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LITTÉRATURE CITÉE

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* Si x_s est une fonction monotone de v [$v = \varphi(u, x_r)$], la recherche de x_s d'après le tableau (C') ne présente pas de difficultés.

** En ce qui concerne les conditions pour qu'une fonction à 3 variables admette une représentation de la forme (8) ou (9), voir [1].

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ASTRONOMY

THE NATURE OF THE SURFACE AND ATMOSPHERE OF THE PLANET MARS ACCORDING TO DATA OF ABSOLUTE PHOTOMETRY

By N. N. SYTINSKAYA

(Communicated by B. G. Fessenkoff, Member of the Academy, 13.X.1943)

The problem of absolute photometry of a planet consists in determining for different points of the planet's disc the value of ρ, which is the ratio of the brightness B at a certain point to the brightness B_0 of an absolutely white orthotropic screen placed normally to the Sun rays at the boundary of the planet's atmosphere

$$\rho = \frac{B}{B_0} \qquad (1)$$

In order to find the value of ρ for the planet Mars I have made use of the photographs taken at different parts of the spectrum during the opposition of 1939 at the Tashkent Astronomical Observatory. These observations have been described in full in a paper by W. Scharonow(1). The photometric standardization of the plates was carried out by the reflecting screen method worked out by myself (2). The screen employed has been examined in detail by L. N. Radlova(3). But this method of standardization could not possibly be used for the ultra-violet part of the spectrum, and I had to resort to an approximate calculation through the brightness of extrafocal star images.

On the basis of these measurements and calculations mean curves were plotted, bringing out the variation of ρ with the angular distance i from the centre of the disc for the bright («continents») and dark («seas») regions of the surface. The data obtained are given in Table 1.

The brightness B of some point of a planet's disc surrounded by an atmosphere is known to be composed of two parts: the light reflected by the surface of the planet and weakened by extinction on passing twice through the atmosphere, and the light scattered in the atmosphere itself. As the optical thickness of the atmosphere of Mars is not large, one may confine oneself in performing the calculation to the scattering of order one. Besides, it is possible to neglect the illumination of the planet surface by the light scattered in the atmosphere, and if we take into consideration the low value of the surface albedo, it appears that the illumination of the atmosphere by the light reflected from it may also be disregarded.

The observations I have used were made about the time of opposition, when the phase angle was near zero. Therefore the angle at which the rays fall upon the surface and that at which they are reflected to the observer may both be taken to be equal to i. On the basis of what has been said here we can express ρ by the following formula:

$$\rho = r\cos^q i \cdot 10^{-2a\sec i} + \varkappa\left[1 - 10^{-2a\sec i}\right] \qquad (2)$$

Table 1

i \ λ	380	430		560		630		730	
		Cont.	Seas	Cont.	Seas	Cont.	Seas	Cont.	Seas
0°	0.100	0.119	0.114	0.184	0.168	0.236	0.163	0.280	0.161
10	0.100	0.119	0.114	0.182	0.166	0.232	0.161	0.272	0.158
15	0.100	0.118	0.113	0.180	0.163	0.227	0.157	0.265	0.156
20	0.099	0.117	0.112	0.174	0.160	0.220	0.153	0.256	0.152
23	0.098	0.117	0.112	0.170	0.157	0.214	0.150	0.249	0.148
26	0.098	0.116	0.111	0.166	0.155	0.209	0.147	0.243	0.146
28	0.098	0.116	0.110	0.164	0.153	0.204	0.141	0.235	0.143
31.3	0.097	0.115	0.110	0.160	0.150	0.196	0.138	0.229	0.140
33.5	0.096	0.115	0.109	0.157	0.147	0.193	0.135	0.222	0.136
36.5	0.095	0.114	0.109	0.151	0.143	0.184	0.128	0.211	0.130
41.5	0.094	0.113	0.108	0.145	0.137	0.171	0.119	0.194	0.122
44.5	0.094	0.112	0.108	0.140	0.134	0.162	0.115	0.183	0.117
48.0	0.093	0.111	0.107	0.135	0.129	0.154	0.108	0.170	0.111
51.5	0.092	0.110	0.107	0.130	0.125	0.145	0.102	0.167	0.105
55.0	0.091	0.109	0.107	0.124	0.121	0.135	0.096	0.154	0.099
59.0	0.090	0.108	0.106	0.119	0.120	0.126	0.092	0.130	0.095

where a is the extinction factor of the planet's atmosphere for the vertical direction; $\varkappa = \frac{\sigma}{2a}$ (σ, the scattering factor of the atmosphere in the direction to the observer); r, the brightness factor of the surface with $i=0$; and the exponent q expresses the law of the reflection of light by the surface. For an orthotropic surface obeying «the cosine law» of Lambert we have $q=1$. For an uneven surface q is below unity; for the Moon, for instance, $q=0$, and in the case of strongly dissected formations, such as forest, bush or meadow, q may be even negative.

On the other hand, in the case of very small lustrous surfaces, q is above unity, and for an ideal mirror $q=\infty$. It appears that q may be appropriately described as «factor of smoothness».

Below we give the values of q for different materials:

Barite screen	+ 0.91
Gypsum screen	+ 0.51
White painted screen	+ 1.11
Opal glass	+ 1.30
Quartz sand (dry)	+ 0.14
Dry loamy soil	+ 0.20
Limestone, even	+ 0.36
» rough	+ 0.07
Brick	+ 0.32
Sod	— 0.05
Moss	— 0.07
Salt crust, crystalline	+ 2.74
» » amorphous	+ 0.81

The figures of Table 1 may be interpreted to mean that taking the quantities a, $\varkappa$, r and q to be invariable with i, one may select such values for them as will best agree with observations. The results of calculations are given in Table 2.

Table 2

Part of spectrum	r			a
	Continents	Seas	Polar caps	
380 mμ	0.057	0.057	0.300	0.089
430	0.092	0.083	0.280	0.076
560	0.190	0.163	0.280	0.045
630	0.238	0.160	0.240	0.035
730	0.283	0.155	0.180	0.022
q	1.220	1.200	1.000	

The parameter $\varkappa$ is put at 0.188, this value corresponding to a scattering obeying Rayleigh's law in the absence of true absorption.

The obtained values of a for different wave-lengths λ fail to correspond to Rayleigh's law λ^{-4}, but are well expressed by the King's formula

$$a = D + \frac{C}{\lambda^4} = 0.0220 + \frac{0.0172}{\lambda^4} \qquad (3)$$

This means that the atmosphere of Mars is not a pure gas, but rather an aerosol consisting of a gaseous medium in which large particles of aerial plankton are held in suspension. If we assume that the atmospheric pressure is determined solely by the gas component of the atmosphere, and that the mass of the gas is proportional to the constant C in King's formula, then from the comparison with the data of Fowle (4) for the Earth atmosphere, we shall find that the mass of the gas per unit area is 3.4 times less on Mars than on the Earth, and consequently, the mercurial barometer there will show a pressure of 220 mm. As the gravity of Mars is 0.38 that of the Earth, the true pressure, *i. e.* the one measured with an aneroid barometer, will be 83.6 mm.

The term D of King's formula expresses the content of large particles, *e. g.* of dust in the atmosphere of Mars. Its magnitude turned out to be of the same order as that for the layer of Earth atmosphere above the level of Mt. Wilson. But the amount of gas being three times less, the dust content of a unit mass of gas must be 2.3 times higher in the atmosphere of Mars than over Mt. Wilson.

The absolute values of the brightness factor r for «the continents» of Mars are close to those found by E. L. Krinov (5) for reddish sands and clays covering the deserts of Central Asia. For «the seas» it is rather difficult to find an analogy among the materials of the Earth. Such substances as grey soil, grey granite and the like fit better than the other.

But none of these materials show the decrease of reflectivity in the right part of the spectrum displayed by «the seas».

«The seas» are by no means areas entirely covered by green plants, as we have it on the Earth, since in this case they would appear darker in the blue rays, and white, in the infra-red rays (the so-called Wood effect). The polar caps can be neither snow nor dense clouds, because these substances reflect light neutrally and have an albedo of order 0.9. Of the Earth's formations ice is nearest to the polar caps in the brightness factor, and this fact confirms the hypothesis of Tikhov (6) put forward in 1909, according to which the polar caps are accumulations of ice analogous to the Earth's glaciers.

The bright spots of the Mars terminator, established by Wright (7) in the blue and violet rays during the opposition of 1924, were observed again in 1939. If we assume that these are regions of a more turbid atmosphere compared to the other parts, and, accordingly, with a large coefficient a, and leave the other parameters of formula (2) without change, we shall obtain the variation of a with angle i along the equatorial diameter of the planet, as given in Table 3 for several different days of July.

We see that the daily course of the extinction factor on Mars is very different in different days. In some cases, as for instance on the 19th of July, the turbidity of the atmosphere was practically constant, but on other days with a low position of the Sun, it was two times higher than at the point under the Sun. Since the increase of turbidity is observed only in the blue and violet rays, but not in the red and infra-red, we cannot escape the conclusion that with a low position of the Sun, light fogs of very small particles giving Rayleigh's scattering appear in the atmosphere of Mars. The ice mist that we sometimes observe on frosty days is perhaps the Earth' analogue of such phenomenon.

The results of the observations described here disprove Wright's hypothesis that Mars possesses an atmosphere of large optical thickness, and are rather in favour of the old assumptions as to the stratospheric climate of this planet.

Table 3

	i	7.VII	10.VII	12.VII	16.VII	19.VII
Morning part	60°	0.078	0.125	0.150	0.120	0.082
	50	0.078	0.124	0.157	0.120	0.080
	40	0.078	0.120	0.155	0.120	0.084
	30	0.078	0.105	0.127	0.118	0.085
	20	0.077	0.092	0.097	0.099	0.085
	10	0.076	0.071	0.080	0.083	0.086
	0	0.076	0.070	0.076	0.077	0.081
Evening part	10	0.075	0.070	0.080	0.078	0.080
	20	0.077	0.076	0.097	0.086	0.078
	30	0.086	0.081	0.105	0.100	0.076
	40	0.097	0.096	0.115	0.116	0.078
	50	0.110	0.110	0.124	0.126	0.083
	60	0.115	0.115	0.116	0.105	0.091

The supposition that the solid surface of Mars is similar to a desert landscape accords well with the optical characteristics obtained for that planet.

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PHYSICS

ON CUMULATIVE EXCITATION OF HYDROGEN AND HELIUM BY ELECTRONIC IMPACT

By B. M. YAVORSKY

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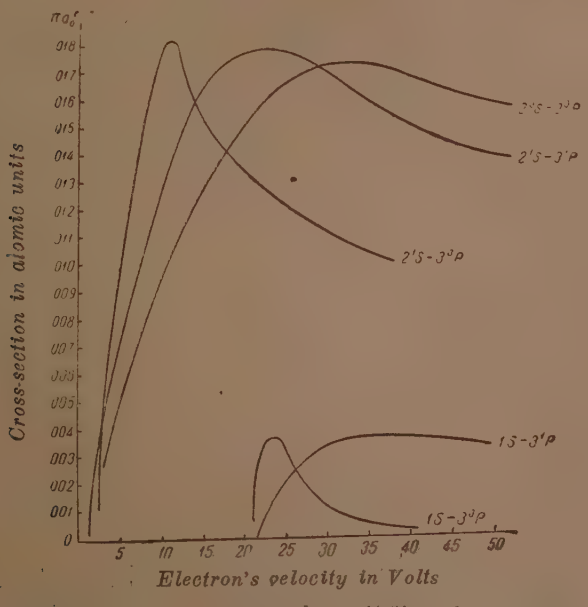
This paper is concerned with consecutive computations of the cross-sections at which certain therms of hydrogen and helium are excited by electronic impact. An arbitrary state S is taken for the initial state of the atom. Following the method of Oppenheimer (1), the interchange effect is allowed



Computed cross-sections for the excitation of some levels of helium atom.

for. The work is a further step in the investigations undertaken by Massey and Mohr (2). They took the unexcited state of the atom for its initial state.

1. Hydrogen Atom. The effective cross-section corresponding to the transition of the atom from state n to state m has the form

$$\Phi = \frac{k'}{k}\,\frac{1}{4}\left\{3[f-g]^2 + [f+g]^2\right\} \tag{1}$$

where

$$f = \sqrt{2\pi}\,\frac{2\pi m}{h^2}\int\int V\psi_n(\vec{r}_1)\,\bar{\psi}_m(\vec{r}_1)\,e^{i(k\vec{n}_0\vec{r}_2 - k'\vec{n}_1\vec{r}_2)}\,d\varrho_1 d\varrho_2 \tag{2}$$

$$g = \sqrt{2\pi}\,\frac{2\pi m}{h^2}\int\int V\psi_n(\vec{r}_1)\,\bar{\psi}_m(\vec{r}_2)\,e^{i(k\vec{n}_0\vec{r}_2 - k'\vec{n}_1\vec{r}_1)}\,d\varrho_1 d\varrho_2 \tag{3}$$

The term f corresponds to Born's theory of collisions ([3]), and the term g allows for the interchange between the impinging and the atom's electron.

Here the subscripts 1 and 2 refer to the atom's and the impinging electron, respectively; V is the energy of interaction; $\vec{n}_0$ and $\vec{n}_1$, the direction of the plane wave of the impinging electron before and after the impact; $\frac{kh}{2\pi m}$ and $\frac{k'h}{2\pi m}$ are the electron's velocities before and after the impact.

For wave functions of the problem we take the symmetrical and antisymmetrical linear combinations of the product of the wave function of the atomic electron by the wave function of the impinging electron

$$\psi_{nk} = \frac{1}{\sqrt{2}}\left\{\psi_n(\vec{r}_1)\,e^{ik\vec{n}_0\vec{r}_2} \pm \psi_n(\vec{r}_2)\,e^{ik\vec{n}_0\vec{r}_1}\right\} \tag{4}$$

$$\psi_{mk'} = \frac{1}{\sqrt{2}}\left\{\psi_m(\vec{r}_1)\,e^{ik'\vec{n}_1\vec{r}_2} \pm \psi_m(\vec{r}_2)\,e^{ik'\vec{n}_1\vec{r}_1}\right\} \tag{5}$$

e and Z have their usual meaning.

The wave functions of a hydrogen atom are set, as ordinarily, according to Schrödinger ([4]).

In computing f the integration over ϱ_2 can be carried out by the method proposed by Bethe ([5]). It then gives

$$\int \frac{e^{i(k\vec{n}_0 - k'\vec{n}_1)\vec{r}_2}}{r_{12}}\,d\varrho_2 = \frac{4\pi}{K^2}\,e^{i(k\vec{n}_0 - k'\vec{n}_1)\vec{r}_1} \tag{6}$$

where

$$K^2 = k^2 + k'^2 - 2kk'\cos\delta \tag{7}$$

$$\delta = \overset{\frown}{n_0 n_1}$$

The expression to be computed has the form

$$f = \alpha N_{n_1 l_1 m_1} N_{n_2 l_2 m_2}\,\frac{4\pi}{K^2}\int \Phi(r_1)\,P_{l_1}^{m_1}(\cos\vartheta_1)\,P_{l_2}^{m_2}(\cos\vartheta_1)\,e^{i\varphi_1(m_1 - m_2)} \times$$
$$\times \exp\left\{ir_1(k\cos\vartheta_1 - k'\sin\vartheta_1\sin\delta\cos\varphi_1 - k'\cos\vartheta_1\cos\delta)\right\} r_1^2\sin\vartheta_1\,dr_1\,d\vartheta_1\,d\varphi_1 \tag{8}$$

where $\alpha = (2\pi)^{3/2}\frac{m}{h^2}$; $N_{n_1 l_1 m_1}$ and $N_{n_2 l_2 m_2}$ are normalizing factors of the wave functions of the hydrogen atom in the initial and final state;

$$\Phi(r_1) = e^{-\frac{Zr_1}{a_0}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}\left(\frac{2Zr_1}{n_1 a_0}\right)^{l_1} L_{n_1+l_1}^{2l_1+1}\left(\frac{2Zr_1}{n_1 a_0}\right)\left(\frac{2Zr_1}{n_2 a_0}\right)^{l_2} L_{n_2+l_2}^{2l_2+1}\left(\frac{2Zr_1}{n_2 a_0}\right) \tag{9}$$

Integration over φ_1 is easily performed by Sommerfeld's method ([6]). In integrating over ϑ_1 allowance is made, in the first place, for the relation existing between spherical and Hugenbauer's functions ([7]).

If what is called a particular case of Hugenbauer's integral ([8]) is applied to integration over ϑ_1, the aim will be attained.

Computing, at last, the integral over r_1, we have to take into consideration that S is the initial state and hence $l_1 = 0$; $P_{l_1}^{m_1}(\cos\vartheta_1) = 1$.

Making use of the expansion in a series of Lagger's polynomials (9) we get for f

$$f=(-1)^{n_1+l_1}A(K,\psi)\left(\frac{n_2a_0}{2Z}\right)^{5/2}\sum_{\tau=0}^{n_1+l_1}(-1)^{\tau}\left(\frac{n_2}{n_1}\right)^{n_1+l_1-\tau}\times$$

$$\times\frac{(n_1+l_1)!\,(2l_1+1+n_1+l_1)!}{\tau!\,(n_1+l_1-\tau)!\,(2l_1+1+n_1+l_1-\tau)!}I_\tau \qquad (10)$$

Here

$$\psi=\arccos\left(\frac{k-k'\cos\delta}{K}\right)$$

$$A(K,\psi)=\alpha N_1N_2\frac{4\pi e^2}{K^2}\sqrt{\frac{2\pi}{K}}\,2\pi i^{l_2}P_{l_2}^{m_2}(\cos\psi) \qquad (11)$$

$$I_\tau=\int_0^\infty e^{-\frac{\xi}{2}\left(1+\frac{n_2}{n_1}\right)}\xi^{l_1+l_2+n_1-\tau+3/2}L_{n_2+l_2}^{2l_2+1}(\xi)\,J_{l_2+1/2}(\zeta\xi)\,d\xi \qquad (12)$$

$$\xi=\frac{2Zr_1}{n_2a_0};\quad \zeta=\frac{Kn_2a_0}{2Z}$$

a_0 is the radius of the first Bohr's orbit of the hydrogen atom.

Finally, in computing (12) the theorem on the expansion of Lagger's polynomials is made use of (9). Then after using the integrals calculated in the theory of Bessel functions and applying the definitions of Hugenbauer's polynomials (8) we may obtain the following result:

$$I_\tau=D_\tau(n_2+l_2)!\,\{(1+n)[\varepsilon^{n_2-l_2-1}C_{n_2-l_2-1}^{l_2+\frac{n_1-\tau}{2}+2}(\varepsilon)\,a_0+\varepsilon^{n_2-l_2-2}C_{n_2-l_2-2}^{l_2+\frac{n_1-\tau}{2}+2}(\varepsilon)\,a_1+\dots$$

$$\dots+\varepsilon^{n_2-l_2-(n_1-\tau+2)}C_{n_2-l_2-(n_1-\tau+2)}^{l_2+\frac{n_1-\tau}{2}+2}(\varepsilon)\,a_{n_1-\tau+1}]+$$

$$+(1-n)[\varepsilon^{n_2-l_2-2}C_{n_2-l_2-2}^{l_2+\frac{n_1-\tau}{2}+2}(\varepsilon)\,a_0+\dots$$

$$\dots+\varepsilon^{n_2-l_2-(n_1-\tau+3)}C_{n_2-l_2-(n_1-\tau+3)}^{l_2+\frac{n_1-\tau}{2}+2}(\varepsilon)\,a_{n_1-\tau+1})]\} \qquad (13)$$

Here

$$D_\tau=\frac{2^{l_2+n_1-\tau+5/2}\,\zeta^{l_2+1/2}\,\Gamma(2l_2+n_1-\tau+3)}{\Gamma(l_2+3/2)\,[(n+1)^2+4\zeta^2]^{l_2+\frac{n_1-\tau}{2}+2}};$$

$$n=\frac{n_2}{n_1};\quad a_k=\frac{(n_1-\tau+1)!\,(-1)^k}{k!\,(n_1-\tau+1-k)!} \qquad (14)$$

$$\varepsilon=\left[\frac{(n-1)^2+4\zeta^2}{(n+1)+4\zeta^2}\right]^{1/2}$$

and the Hugenbauer polynomials (8) are determined as usual

$$(1-2tu+u^2)^{-\nu}=\sum_{s=0}^{\infty}C_s^\nu(t)\,u^s \qquad (15)$$

For calculating the interchange term g we need not alter our choice of coordinate axes, and the expression for $\frac{1}{r_{12}}-\frac{Z}{r_2}$ opens in the following form:

$$\sum_{\nu=0}^{\infty}\sum_{p=-\nu}^{\nu}\gamma_\nu(r_1,r_2)\,P_\nu^p(\cos\vartheta_1)\,P_\nu^p(\cos\vartheta_2)\cos p(\varphi_1-\varphi_2) \qquad (16)$$

where

$$\text{for } \nu=0\begin{cases}\gamma_0=\dfrac{1-Z}{r_2}, & r_1<r_2\\ \gamma_0=\dfrac{1}{r_1}-\dfrac{Z}{r_2}, & r_1>r_2\end{cases}\qquad \text{for } \nu\neq 0\begin{cases}\gamma_\nu=\dfrac{r_1^\nu}{r_2^{\nu+1}}, & r_1<r_2\\ \gamma_\nu=\dfrac{r_2^\nu}{r_1^{\nu+1}}, & r_1>r_2\end{cases}$$

Integration over φ_2, φ_1 and ϑ_1 is carried out in a similar manner. Integration over ϑ_2 is performed by developing $e^{ikr_2\cos\vartheta_2}$ in zonal harmonic waves ([11]) and applying Gaunt's formula ([11]) for the integration of a product of three spherical functions.

As the interchange term is essential only in the region of small velocities, its exact value in this region may be obtained by using the asymptotic representation of Bessel functions ([12]). The computation of the integrals over r_1 and r_2 is thereby considerably simplified, and the interchange term is obtained in the form

$$g=\frac{(-1)^{n_1+n_2+l_2}\,2^{n_1+n_2+2l_2+1}\,(l_2-m_2)!\,\pi^3 a e^2 N_{n_1 00}N_{n_2 l_2 m_2}n_1!\,(n_1+1)!\,(n_2+3l_2+1)!}{\left(\frac{Z}{n_1a_0}\right)^2\left(\frac{Z}{n_2a_0}\right)^2}\times$$

$$\times\sum_{\nu=m_2}^{\infty}\frac{P_\nu^{m_2}(\cos\vartheta)\,i^\nu}{\Gamma(\nu+{}^3/_2)}(m_2+\nu)!\,k'^\nu\times$$

$$\times\sum_s{}'\sum_t{}''\frac{(-1)^{n-\nu+t}\,k^s\,(s!)^2\,u!\,(2u-2s)!\,(l_2+m_2+t)}{\left(\frac{Z}{n_1a_0}\right)^s\Gamma(s+{}^3/_2)\,(u-l_2)!\,(u-\nu)!\,(u-s)!\,(2u+1)!}\times$$

$$\times\frac{(\nu+s-m_2-t)!\,I_{\tau,\tau',s,\nu}}{(l_2-m_2-t)!\,(\nu-s+m_2+t)!\,t!\,(s-t)!}\qquad(17)$$

where $u={}^1/_2(l_2+\nu+s)$, l_2, ν and s are such that $l_2+s>\nu$, and summation over t is carried on until one of the factorials becomes negative ([11]). $I_{\tau,\tau',s,\nu}$ is a double sum of the form

$$I_{\tau,\tau',s,\nu}=\sum_{\tau=0}^{n_1}\sum_{\tau'=0}^{n_2+l_2}(-1)^{\tau+\tau'}\frac{1}{\tau!\tau'!2^\tau 2^{\tau'}(n_1-\tau)!\,(n_1+1-\tau)!\,(n_2+l_2-\tau')!\,(n_2+3l_2+1-\tau')!}\times$$

$$\times\left[\frac{(\nu+n_1-\tau+1)!\,(s+2l_2+n_2+2-\tau')!}{\left(\frac{Z}{n_1a_0}\right)^\nu\left(\frac{Z}{n_2a_0}\right)}-\frac{Z\,(\nu+2+n_1-\tau)!\,(s+2l_2+n_2+1-\tau')!}{\left(\frac{Z}{n_1a_0}\right)^{\nu+1}}+\right.$$

$$\left.+\frac{(2\nu+2+n_1-\tau)!\,(2l_2+n_2+s-\nu+1-\tau')!}{\left(\frac{Z}{n_1a_0}\right)^{2\nu+1}\left(\frac{Z}{n_2a_0}\right)^{-\nu}}+\frac{(n_1-\tau+1)!\,(2l_2+n_2+s+\nu+2)!}{\left(\frac{Z}{n_2a_0}\right)^{\nu+1}}\right]\qquad(18)$$

The total effective section for the excitation of the hydrogen atom after integration over scattering angles takes the form

$$\Phi=\int_0^\pi\sum_{m_2=-l_2}^{l_2}\frac{k'}{k}\{3(f-g)^2+(f+g)^2\}\sin\delta\,d\delta\qquad(19)$$

2. Helium Atom. Several authors ([13]) have dealt with the problem of wave functions of singlet and triplet therms of helium. These functions are

well known for $1S$, 2^1S, 2^1P and 2^3P levels, and for higher levels, *e. g.* $3P$ and $3D$.

In order to calculate the effective cross-section one has to compute the expressions

$$f = \alpha \int\int\int \left(\frac{2}{r_3} - \frac{1}{r_{23}} - \frac{1}{r_{13}}\right) \psi_n(r_2, r_1)\, e^{i(k\vec{n}_0\vec{r}_3 - k'\vec{n}_1\vec{r}_3)}\, \bar{\psi}_m(r_2, r_1)\, d\varrho_1\, d\varrho_2\, d\varrho_3 \qquad (20)$$

$$g = \alpha \int\int\int \left(\frac{2}{r_3} - \frac{1}{r_{23}} - \frac{1}{r_{13}}\right) \psi_n(r_2, r_1)\, e^{i(k\vec{n}_0\vec{r}_3 - k'\vec{n}_1\vec{r}_2)}\, \bar{\psi}_m(r_1, r_3)\, d\varrho_1\, d\varrho_2\, d\varrho_3 \qquad (21)$$

The wave functions of the excited states of helium are known ([14]). Therefore, when S is the initial state, the integrals are computed by the previous methods ([2]). The computation of the term

$$\int\int\int \frac{1}{r_3} \psi_n(r_1, r_2)\, \bar{\psi}_m(r_1, r_3)\, e^{ik\vec{n}_0\vec{r}_3 - ik'\vec{n}_1\vec{r}_3}\, d\varrho_1\, d\varrho_2\, d\varrho_3$$

for instance, leads to the following type of integral over the coordinates of electron 2

$$\int e^{ik'\vec{n}_1\vec{r}_2}\, e^{-\mu r_2}\, r_2^s P_{l_2}^{m_2}(\cos\vartheta_2)\, e^{im_2\varphi_2} \sin\vartheta_2\, d\vartheta_2\, d\varphi_2\, dr_2$$

If the axes are defined in the same way as in the one-electron problem, one may obtain

$$\int\int\int \exp\{ik' r_2 (\sin\vartheta_2 \cos\varphi_2 \sin\delta + \cos\vartheta_2 \cos\delta)\} \times$$
$$\times\, e^{-\mu r_2} r_2^s P_{l_2}^{m_2}(\cos\vartheta_2)\, e^{im_2\varphi_2} \sin\vartheta_2\, d\vartheta_2\, d\psi_2\, dr_2$$

and this integral can be reduced ([2]) to

$$2\pi i^{l_2} \sqrt{\frac{2\pi}{k}}\, P_{l_2}^{m_2}(\cos\delta) \int_0^\infty e^{-\mu r_2} r_2^{s-1/2} J_{l_2-1/2}(k' r_2)\, dr_2 =$$

$$= \frac{(2\pi)^{3/2} i^{l_2} P_{l_2}^{m_2}(\cos\delta)\, k'^{l_2} \mu \Gamma(l_2 + s + 1)}{2^{l_2+1/2} \mu^{l_2+s+1} \Gamma(l_2 + 3/2)} F\left(\frac{l_2+s+1}{2}, \frac{l_2+s+2}{2}, l_2 + 3/2, -\frac{k'^2}{\mu_2}\right) \qquad (22)$$

F is a hypergeometrical function which degenerates here into an elementary function. The total effective section integrated over scattering angles assumes the form

$$\int_0^\pi \sum_{m=-l_2}^{l_2} \left[f_{n_2, l_2}^m(kk') P_{l_2}^m(\cos\psi) - \sum_{\nu=m}^{\infty} P_\nu^m(\cos\delta)\, g_{\nu, l_2, n_2}^m(kk') \right]^2 \sin\delta\, d\delta \qquad (23)$$

Upon a cumulative excitation of helium the excitation potentials correspond to that region of electron velocities in which the interchange effect plays the principal part. The region of small velocities of the exciting electrons, which is practically important in the discharge, corresponds to the optimum conditions of cumulative excitation. If in the expression for the effective cross-section we are to retain the interchange term alone and compute all those terms of the series which present a decrease by two orders of magnitude, it will be possible to obtain results for some simplest therms.

Direct experimental investigations of the step excitation have not been made. Effective sections of helium have been studied by the optical method([15]).

From the computations performed it is seen that the effective cross-section for cumulative processes is one order of magnitude higher than that for direct excitation.

The available experimental data may indirectly be used as a check of the performed computations.

Thus, the experimental and theoretical curves of the functions of excitation correspond to each other in form. Then, applying the results obtained for the particular case of not excited initial state, one obtains curves coinciding with the investigations of Massey and Mohr.

An interesting case is the curve of the excitation function for the triplet therm when the initial state is a triplet state too. We find this curve to depart from the form ordinarily corresponding to a triplet and assume that corresponding to a singlet.

I wish to express my gratitude to V. Fabrikant, who, having suggested this investigation, took interest in its progress and gave many a valuable advice.

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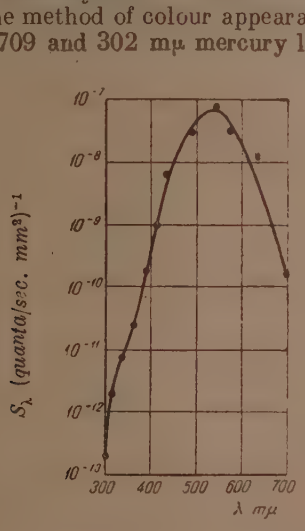
BIOPHYSICS

ABSOLUTE PHOTOPIC SENSITIVITY OF THE EYE IN THE ULTRA-VIOLET AND IN THE VISIBLE SPECTRUM

By N. I. PINEGIN

(Communicated by S. I. Wawilow, Member of the Academy, 24.XI.1943)

In an earlier paper of mine published in 1941 (1) I reported some measurements of photopic and scotopic sensitivity made on 9 observers after one hour adaptation to the dark, according to the method of colour appearance. The measurements were made within the range of 709 and 302 mμ mercury lines.



In this paper we consider the question of absolute photopic sensitivity, as a value, reverse to the ilumination of the retina $S_\lambda = 1/I_\lambda$.

The absolute photopic sensitivity S_λ [expressed as a number of (quanta/sec. sq. mm)$^{-1}$] is related to the illumination of the pupil E_λ (expressed as a number of erg/sec. sq. cm) by the equation

$$S_\lambda = \left[\frac{E_\lambda}{h\nu}(1-r)\frac{\sigma}{a}\right]^{-1} \qquad (1)$$

where r represents the reflection losses at the cornea of the eye (assumed to be 0.05); σ, the area of the pupil (assumed to be 0.33 cm²); and a, the area of the image on the retina. The area of the image was 0.0083 mm².

The adjoined table and figure show the results of our measurements of photopic sensitivity.

The Illumination of the Pupil, the Retinal Illumination and the Absolute Photopic Sensitivity of the Normal Eye in the Ultra-violet and in the Visible Spectrum
(the average values for 9 observers)

λ, mμ	E_λ $\frac{erg}{sec.\ cm^2}$	I_λ $\frac{quanta}{sec.mm^2}$	$S_\lambda \left(\frac{quanta}{sec.mm^2}\right)^{-1}$	log S_λ
302	8.47×10^{-1}	4.63×10^{12}	2.16×10^{-13}	—12.7
313	8.86×10^{-2}	5.34×10^{11}	1.87×10^{-12}	—11.7
334	2.13×10^{-2}	1.29×10^{11}	7.75×10^{-12}	—11.1
365	6.69×10^{-3}	4.45×10^{10}	2.25×10^{-11}	—10.6
390	6.84×10^{-4}	4.76×10^{9}	2.10×10^{-10}	— 9.7
404—407	1.36×10^{-4}	1.00×10^{9}	1.00×10^{-9}	— 9.0
435	1.88×10^{-5}	1.49×10^{8}	6.71×10^{-9}	— 8.2
491	3.35×10^{-6}	2.99×10^{7}	3.34×10^{-8}	— 7.5
546	1.34×10^{-6}	1.33×10^{7}	7.52×10^{-8}	— 7.1
576—579	2.74×10^{-6}	2.88×10^{7}	3.47×10^{-8}	— 7.5
690—709	4.91×10^{-4}	6.23×10^{9}	1.60×10^{-10}	— 9.8

The curve of the photopic sensitivity has its maximum at about 546 mμ.

A more exact determination of the maximum does not appear to be obtainable, as the mercury spectrum has no corresponding lines. In the afore-mentioned paper(2) we wrote: «the chromatic sensitivity decreases regularly with the decrease of the wave-length to 302 mμ». But speaking more strictly, the photopic sensitivity curve in the 365—334 mμ region has a small but still visible flattening, after which it drops abruptly again.

The curve of scotopic sensitivity in this region has a maximum (in the case of some observers) or a pronounced flattening (in other cases).

Apparently some maximum of transmission in the lens takes place in the region of 365—334 mμ.

Measurements of the photopic sensitivity curve in the ultra-violet were made by C. F. Goodeve (unpublished). These measurements clearly show a marked flattening of the curve and a point of inflection at about 350 mμ.

Thus, our results as concerns the character of the curve in the ultra-violet, find confirmation in the work of the above-mentioned author.

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CHEMISTRY

COMPOSITION OF INTERMETALLIC COMPOUNDS AS DETERMINED FROM IONIZATION POTENTIALS OF THEIR COMPONENTS

By O. E. ZVIAGINCEV

(Communicated by I. I. Černjaiev, Member of the Academy, 28.XII.1943)

In 1942 I published a paper (1) in which it was proved that intermetallic compounds are formed by metals whose ions have close energy coefficients [VEK's, to use the symbol suggested by A. E. Fersman (2)].

This time my object has been to attempt a theoretical computation of the probable composition of binary intermetallic compounds on the basis of the physical values characterizing their components. In this I deliberately confine myself to the most simple cases.

In accordance with the school of N. S. Kurnakov, I mean by intermetallic compounds those binary alloys which give singular points on the composition—property curves, and have moreover an ordered crystalline structure (3), *e. g.* CuAu, MnAu, Mg_2Sn and others. These are commonly described as daltonides.

In the class of intermetallic compounds belong also those intermediate phases (bertollides, according to N. S. Kurnakov) which have no singular points upon the composition—property diagram, because they are dissociated not merely in the liquid but also in the solid phase (4). β-,γ- and σ-phases in the system copper—zinc, and γ-phase in the aluminium—magnesium system may serve as an example of such intermediate phases. These phases form typical crystalline lattices.

Most of the intermetallic compounds are characterized by the occurrence of maxima on the composition—electroconductivity curves.

Thus, the problem of calculating the probable composition of intermetallic compounds is reduced to the calculation of the position of maxima on the electroconductivity curve. For this purpose I use the theory of electroconductivity of metals as evolved in the modern physics of metals (5, 6).

The electroconductivity of metals is very high at low temperature and declines as the temperature is increased. This is because the thermal oscillations of metal atoms present obstacles to the movement of «electronic gas». Similar obstacles for electrons arise from the presence of admixtures in metals that are not pure. This accounts for a metal decreasing in electroconductivity when another metal is added to it as admixture.

The mechanism underlying the inhibition of electronic motion by admixtures may be imagined as follows. Different metals have different ionization potentials* (energies necessary for severing the ion from the atom). If the ionization potential be denoted by J, the atomic volume of the metal by V, then the ratio J/V which may be called atomic concentration of the ionization energy, will be different for different metals. In case metal A contains metal B dissolved in it as admixture,

* The ionization potential is connected with the VEK 's of Fersman by a linear relation.

there are places created in the crystalline lattice where $\frac{J_A}{V_A} > \frac{J_B}{V_B}$: this restrains the movement of electrons, whereby the electroconductivity of the metal is reduced.

In a bi-metallic alloy this phenomenon should apparently and does actually take place. When one metal is dissolved in another, nodes are created in the crystalline lattice of the alloy, having values of the J/V ratio different from the main metal. Owing to this, solid solutions are always less conductive than pure metals.

The study of binary metallic solutions has shown that intermetallic compounds have in the majority of cases an increased electric conductivity as compared to that of solid solutions. Thus, at points corresponding to intermetallic compounds the motion of electrons is not so strongly restrained. This circumstance has led me to suppose that in the intermetallic compound under consideration the energy concentrations J/V of the two metals become equal.

If we denote by n the atomic per cent of the component A in the alloy, then the content of the second component will be $100-n$ per cent. The concentration of the ionization energy will be $\frac{J_A}{\frac{n}{100} V_A}$ in the first metal and $\frac{J_B}{\left(\frac{100-n}{100}\right) V_B}$ in the second. For n corresponding to an intermetallic compound with a maximum conductivity, the concentration of the ionization energy of the two components will equal:

$$\frac{J_A}{nV_A} = \frac{J_B}{(100-n)V_B} \qquad (1)$$

This equation sets one of the conditions for the existence of intermetallic compound.

If the components and their ionization potentials are known, the probable composition of the compound may be derived from (1) by determining n:

$$n = \frac{J_A V_B \cdot 100}{J_B V_A + J_A V_B} \text{ at. per cent} \qquad (2)$$

In case the metal has several ionization potentials (*e. g.* Al′, Al″, Al‴), they have to be substituted in succession in (2).

The number of intermetallic compounds calculated according to (2) is obviously much higher than their actual number. From the calculated results one has to exclude:

a) The compounds belonging to unstable or scarcely probable ions. Such compounds, for instance, as Tl′, Au″, Mg‴ are very unlikely. In general, ions with a charge higher than $+4$ need not be considered, it seems.

b) The results that fail to satisfy the requirements of crystallo-chemistry. One has, for instance, to exclude the compositions in which the number of atoms of one component very greatly exceeds the number of atoms of the other component (*e. g.* above 8 : 1). We must also omit complicated combinations lying far away from simple stocheiometric atomic relations. In many cases the atoms of two elements taken in a certain numerical ratio cannot be arranged within a stable crystalline lattice, and we have to discard all such cases in which these ratios do not come under the rules formulated for alloys by Goldschmidt (7) and Hume-Rothery(8).

On the above-mentioned considerations we have to discard the compounds $HgNa_{24}$ and $HgNa_9$. The rest of the computed formulae agree satisfactorily with those established by experiment (see example Hg—Na, p. 161).

System Hg—K (p. 161). Here too, the values known from experiment are in good agreement with the computed values. The exception is with the compound Hg_2K which was not predicted by the theory.

According to computations, the compounds Ag—Tl should correspond to 4.6, 15.3, 18.7 and 46 atomic per cent of thallium. If the compound Tl″ is discarded as an

EXAMPLES OF CALCULATIONS OF PROBABLE COMPOSITION OF INTERMETALLIC COMPOUNDS

System: Mercury—Sodium

Hg, ionization potentials 10.39; 29; 65; atomic volume 14.1

Na, ionization potentials: 5.116; 52; atomic volume 23.7

Computed		Observed [10]	
At. % Hg	Formula	At. % Hg	Formula
9	$HgNa_9$	—	—
25	$HgNa_3$	25	$HgNa_3$
48.4	HgNa	50	HgNa
69	Hg_2Na	67.8	Hg_2Na
77.5	$Hg_3(_4)Na$	80	Hg_4Na
96	$Hg_{24}Na$	—	—

System: Mercury—Potassium

Hg, ionization potentials: 10.39; 29; 65; atomic volume 14.1

K, ionization potentials: 4.32; 36; atomic volume 45.5

Computed		Observed	
At. % Hg	Formula	At. % Hg	Formula
95	—	90	Hg_9K
90	Hg_9K	83	Hg_3K
86	Hg_5K	80	Hg_4K
72	Hg_3K	75	Hg_3K
—	—	67.8	Hg_2K
48	HgK	50	HgK

System: Cadmium—Magnesium

Cd, ionization potentials: 8.75; 25.8; atomic volume 13.0

Mg, ionization potentials: 7.61; 22.6 atomic volume 22.6

Computed		Observed	
At. % Cd	Formula	At. % Cd	Formula
28	$CdMg_3$	25	$CdMg_3$
53.7	CdMg	50	CdMg
55	?	67.8	$CdMg_2$
79	$CdMg_3$	75	$CdMg_3$

System: Gold—Copper

Cu, ionization potentials; 7.691; 25.8; atomic volume 7.0

Au, ionization potentials: 9.25 (30); (65); atomic volume 10.2

Computed		Observed [14]	
At. % Cu	Formula	At. % Cu	Formula
15	$CuAu_6$	—	—
36	$CuAu_3$ or Cu_2Au_3	—	—
55	CuAu	50	CuAu
64.8	Cu_2Au	75	Cu_3Au

System: Silver—Thallium

Ag, ionization potentials; 7.54; 25.5; atomic volume 10.2

Tl, ionization potentials: 6.08; 29.06; atomic volume 45.4

unstable ion, and the compound $TlAg_5$ because of the impossible volume ratio of its components, we shall find that Ag and Tl should not be expected to form compounds, and in fact they do not form any [13].

The compound $CuAu_6$ is rather improbable from the viewpoint of volume relations. The compound $CuAu_2$ (or Cu_2Au_3) is probable, but has not yet been found. The remaining two cases present coincidences (although not very accurate ones) of the calculated and known composition. The compound Au_2Cu_3 whose existence is in doubt, has not been foreseen by me theoretically.

System: Gold—Manganese

Au, ionization potentials: 9.25; (65); atomic volume 10.2

Mn, ionization potentials: 7.40; 23.10; 55.107; atomic volume 7.6

Computed		Observed [15]	
At. % Mn	Formula	At. % Mn	Formula
13.2	Au_6Mn	—	—
32.8	Au_2Mn	33.3	Au_2Mn
51.7	AuMn	50	AuMn
52.7	AuMn		
69	$AuMn_3$	75	$AuMn_3$
77	$AuMn_3$		
89	$AuMn_9$	—	—
94	$AuMn_{20}$	—	—

The ionization potentials for Au''' are known only approximately. Au'' was discarded by me as unstable.

The results of computation of the system Au—Mn agree well with the experimental data ([15]). Apparently, the extreme values of Au_6Mn, $AuMn_9$ and $AuMn_{20}$ may be omitted, because they fail to satisfy the conditions (2).

Lack of space precludes any further reference to other systems for which good agreement of theory and experiment has been obtained (*e. g.* Ag—Al, Au—Zn, Ni—Mg, *etc.*).

In deriving (2) I deliberately made considerable simplifications, taking no account of phenomena arising from the mutual influence of ions, though they undoubtedly affect the values of ionization potentials (sometimes by as much as 5, 10 and more per cent) and even the atomic volume. For more precise computation corresponding corrections must be introduced in formula (2).

Neither have I made allowance for the difference of temperature which affects the atomic volume. Since in exact calculation only the difference in the dilatation coefficients of the two components is of importance and not the absolute value, this influence is very slight and may be discarded in first approximation. Finally, equation (1) and formula (2) refer only to the ratio between the quantities J and V of the components, without taking account of their absolute values, which appear to have a direct bearing upon the energy of the lattice.

Thus, my computations should be considered only as a rough (first) approximation failing to take many physical factors into account.

Conclusions. A method has been suggested for approximate computation of the composition of intermetallic compounds from atomic volumes and ionization potentials of the components.

Several examples have been given to show the agreement of the calculated results with the experiment.

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GEOCHEMISTRY

ON THE CONTENT OF RUBIDIUM IN PLANTS. I

By T. F. BOROVICK-ROMANOVA

(Communicated by W. I. Vernadsky, Member of the Academy, 24.XII.1943)

The presence of rubidium in plants was reported in 1862 by Grandeau [1] who declared that the amount of this element extracted from the soil by the beet is important and should be given attention. In co-operation with Lefebvre [2], Grandeau isolated rubidium from lyes remaining after the treatment of beet. In his report delivered in 1862 at the Paris Academy, Grandeau said that he had already received 400 g of pure salt of rubidium chloride.

Pfeiffer [3] (1872) and Lippmann [4] (1888) also found rubidium in beet.

Later on this element has more than once been discovered in number of plants: In tobacco leaves, tea, coffee, grapes, in various fruits, vegetables, arborial plants, algae [1,5–13].

In most of the cases the determinations were qualitative, while quantitative determinations were few in number and not all of them were reliable.

The present work was devoted to the determination of rubidium in sea-weeds and sweet-water plants, made by the method of spectrum analysis. The plants were preliminarily incinerated in a muffle at a temperature of 450—500° C. Control tests have shown that this procedure was attended by no appreciable losses in rubidium. As a source of excitation of the spectrum a c. c. arc with coal electrodes was used. The samples to be tested were placed into a depression in the bottom electrode, 5 mm deep. In order to make the conditions of evaporation identical, both the samples and the specimens investigated were added Na_2SO_4 at a ratio 1 : 1. As standards served mixtures approaching by composition that of the plants, which were added definite amounts of rubidium.

The content of rubidium was evaluated by the method of «internal standards» which consisted in that both to the samples tested and the standards were admixed equal amounts of a definite substance, which in our case was Ba, after this comparison was made between the relative intensities of the lines of Rb and Ba in the samples and standards. The relation between the intensities of the lines of Rb and Ba was determined with the aid of a Zeiss microphotometer. The data obtained for the standards were arranged into a graduated graph, according to which the content of rubidium could be determined in the samples investigated. The mean error was ± 12 per cent.

The data given in Tables 1 and 2 permit of the following conclusions.

The content of Rb in sea-weeds is more than tenfold that in the sea water [2×10^{-5} per cent ([14])]. Sea-weeds of the order *Laminariales* are richer in Rb than *Fucales* ($3.2 \times 10^{-4} - 5.6 \times 10^{-4}$ per cent against $1.2 \times 10^{-4} - 1.9 \times 10^{-4}$ per cent). The latter show also a lower K content.

The ratio Rb/K it somewhat higher in algae of the order *Laminariales* than in *Fucales*. On the average, the ratio Rb/K for sea-algae is 37×10^{-5}.

Table 1

Sea Algae

Order, family, genus and species	Place of sampling	Rubidium content in % to ash	Rubidium content in % to live weight	Rb/K ratio
Phaeophyceae				
Order *Laminariales*				
1. *Macrocystis pirifera.*	The Pacific	1.2×10^{-2}	5.6×10^{-4}	—
2. *Laminaria digitata*	The Barents Sea, Daln. Selenzy (Murmansk biol. station, 1938)	1.1×10^{-2}	5.1×10^{-4}	57×10^{-5}
3. » *saccharina*	Do	0.91×10^{-2}	3.3×10^{-4}	37×10^{-5}
4. » *saccharina (linearis)*	»	0.97×10^{-2}	4.4×10^{-4}	—
5. *Alaria esculenta*	»	0.97×10^{-2}	4.6×10^{-4}	—
6. *Phyllaria dermatodea*	»	0.82×10^{-2}	3.2×10^{-4}	—
Order *Fucales*				
7. *Fucus serratus*	The Barents Sea, Ekaterininsk bay, 1933	0.29×10^{-2}	1.9×10^{-4}	—
8. » »	The Barents Sea, Daln. Selenzy (Murmansk biolog. station, 1938)	0.27×10^{-2}	1.4×10^{-4}	21×11^{-5}
9. *Fucus inflatus*	Ekaterininsk bay, 1933	0.32×10^{-2}	1.3×10^{-4}	—
10. *Ascophyllum nadosum*	Do	0.20×10^{-2}	—	—
11. » »	The Barents Sea, Daln. Selenzy (Murmansk biolog. station, 1938)	0.19×10^{-2}	1.2×10^{-4}	27×10^{-5}
Rhodophyceae				
Order *Rhodymeniales*				
12. *Rhodymenia palmata*	The Barents Sea (Murmansk biolog. station)	1.4×10^{-2}	4.3×10^{-4}	—
Order *Bangiales*				
13. *Porphyra laciniata* f. *umbilicalis*	Daln. Selenzy (Murmansk biolog. station)	0.52×10^{-2}	1.8×10^{-4}	—
Chlorophyceae				
Order *Ulotrichales*				
14. *Ulva lactuca*	The Pacific	1.3×10^{-2}	6.6×10^{-4}	—
Order *Charoles*				
15. *Chara hispida*	The Caspian Sea	—	—	—
Higher Sea-Plants				
Fam. *Potamogetonaceae*				
16. *Zostera nana*	The Caspian Sea, 1935	0.32×10^{-2}	1.4×10^{-4}	—
17. *Ruppia spiralis*	The Caspian Sea	0.38×10^{-2}	1.6×10^{-4}	—

In sweet-water plants the content of Rb makes up 1.4×10^{-4} — 5.2×10^{-4} per cent. Considering that the content of Rb in sweet-water is equal to 10^{-7} per cent, it is evident that rubidium is being concentrated in those plants in amounts 1000 times as large as in the surrounding medium. The ratio Rb/K for sweet-water plants is on the average 78×10^{-5}.

Table 2

Sweet-Water Plants

Family, genus, species	Place of sampling	Rb content in % to ash	Rb content in % to live weight	Rb/K ratio
Hydrocharitaceae				
1. *Helodea canadensis* Rich.	Staroselje, Reserve of the Ukrainian Academy 1936	0.82×10^{-2}	2.3×10^{-4}	7×10^{-5}
2. *Stratiotes aloides* L.	Do	1.4×10^{-2}	2.6×10^{-4}	89×10^{-5}
Ceratophyllaceae				
3. *Ceratophyllum demersum* L.	»	1.1×10^{-2}	1.6×10^{-4}	5×10^{-5}
Ranunculaceae				
4. *Ranunculus divalicatus* Koch. (Syn. *Ranunculus circinnatus* Sibth.)	Zvenigorod, 1936	0.96×10^{-2}	4.7×10^{-4}	—
Potamogetonaceae				
5. *Potamegeton luceus* L.	Staroselje, Reserve of the Ukrainian Academy, 1936	5.2×10^{-2}	1.4×10^{-4}	—
Nymphaeaceae				
6. *Nymphaea alba* L. (stalks)	Do	3.3×10^{-2}	—	—
7. *Nymphaea alba* L. (roots)	»	7×10^{-2}	—	—
8. *Nuphar luteum* Sm. (stalks)	»	2×10^{-2}	2.6×10^{-4}	100×10^{-5}
9. *Nuphar luteum* Sm. (roots)	»	6.7×10^{-2}	5.2×10^{-4}	—
Alismataceae				
10. *Sagittaria sagittifolia* L.	Zvenigorod, 1936	1.5×10^{-2}	2×10^{-4}	—
Lemnaceae				
11. *Lemna minor* L.	Do	2.1×10^{-2}	2.7×10^{-4}	—
12. » *trisulca* L.	»	3.1×10^{-2}	4.1×10^{-4}	

I wish to express here my sincere gratitude to W. I. Vernadsky, Member of the Academy, and to Professors A. P. Vinogradov and S. A. Borovick for their valuable suggestions and for the help they kindly lent me in the course of this work.

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GENETICS

AN ATTEMPT OF SYNTHESIZING WINTER WHEATS FOR THE SUB-TAYGA ZONE OF WEST SIBERIA

By A. J. KUPZOW

(Communicated by A. A. Richter, Member of the Academy, 13.XI.1943)

There are still no varieties of winter wheats that would satisfy the economical requirements of the sub-tayga zone of West Siberia and most of the regions of that part of the USSR either. In spite of the successful attempts that point to the possibility of obtaining here yields of this culture, it invariably proved inferior in this respect to the summer wheats.

It is obvious that we possess no ecotype of a Siberian sub-tayga wheat, while attempts to introduce ecotypes from other zones proved here a failure. The so-called «indigenous» Siberian winter wheats are mostly nothing else than varieties imported to Siberia from the European part of the USSR.

Under conditions of comparative variety tests they give a yield that is clearly below that of varieties obtained by breeding; they scarcely present any advantage as to winter hardiness, or deserve serious attention as original material for breeding.

It stands to reason that in order to solve the problem of cultivating winter wheat in Siberia, indigenous ecotypes of this crop should be created. And one of the easiest procedures to succeed in this attempt is by way of synthesis of the ecotypes desirable in the process of hybridizaton.

The rather severe wintering conditions that prevail in Siberia and especially the unfavourable sharp temperature fluctuations during snow melting and early in spring are a feature in common with the steppes of the Volga region; this is why the Volga varieties of winter wheats are the most winter-hardy when transferred to Siberia.

But high winter-hardiness alone does not guarantee for high yields. Xeromorphous steppe wheats do not profit to full extent of the humid summer at the sub-tayga zone of Siberia. In spite of their lower hardiness more hydrophilous types of the winter wheats of the wood-steppe and woody zone of the European part of this country often prove more productive here than the steppe varieties (Table 1).

Besides, these varieties are more resistant against rotting, which is one of the elements of the complex of winter-hardiness in the sub-tayga zone. In winter 1940—41 the steppe varieties showed themselves obviously inferior in this respect to the hydrophilous varieties.

According to their mode of wintering, the first places have been won when tested near Tomsk by the varieties of the Moscow breeding station (1938 and 1940), Saratov station (1939, 1941), or of the Novo-Urensk station (1942).

Table 1

Relative Winter-Hardiness and Productivity of Steppe Winter Wheats and of Varieties from the Wood-Steppe and Woody Zone

(Tested at the Tomsk station for grain crops)

Year	Steppe near-Volga wheats *		Wheats of wood-steppe zone of Volga and Ukraine **		Wheats of woody zone of Central Russia ***	
	yield of grain, centner/ha	overwintering plants in %	yield of grain, centner/ha	overwintering plants, in %	yield of grain, centner/ha	overwintering plants, in %
1938	11.9	44.9	16.6	43.8	17.5	38.8
1939	12.1	67.3	13.6	65.2	12.8	58.4
1940	14.2	66.5	5.2	25.6	14.1	38.0
1941	2.5	18.8	3.6	20.3	3.4	24.8
1942	8.3	33.9	9.4	29.6	7.1	31.7
Average for 5 years	9.8	46.3	9.7	36.9	11.0	38.3

* Saratov varieties: 0329, 434/154, 1060/10, 46/131, and Besenchuk varieties: 072-01, BG-18.

** Kharkov varieties: 0917, 1239, Petrovskaya 122/76, Simbeleyskay 0146, and Novourensk varieties: 122 and Uljanovka.

*** Moscow varieties: 24-11, 02460, 24-53, and Ivanovsky Durable.

It thus becomes quite evident that the ecotype of sub-tayga Siberian wheat should combine the adaptation to continental wintering conditions of the steppe wheats with the hygromorphism of the ecotypes of the woody zone, the latter character permitting the plant to profit of the humid summer, and precluding the possible injury caused by water excess.

The method of hybridization of hygrophilous northern ecotypes with southern xerophilous ones as practised by Ch. Saunders and V. Pissarev, proved highly promising in creating early hygromorphic ecotypes of summer wheats for Canada[1] and for the northern regions of this country[2]. The present author made an attempt to apply it to the breeding of Siberian ecotypes of winter wheats to be grown in the sub-tayga region. At the same time he tried to obtain hybrids between step winter wheats and semi-arid and high altitute ecotypes, in order to reinforce in the progeny the resistance to sharp temperature fluctuations in spring.

In summer 1940 the Saratov variety 0329 was crossed and back-crossed with Dutch winter wheat, with the Moscow variety 24-11, Uzbeck winter wheat, the Pamir summer variety Ezhovka and with Garnet. The components were being grown and the crosses practised in a cool greenhouse. The F_1 was grown in 1941 under similar conditions, the progeny of both winter parents was sown on February 1, while in those cases when one of the parents was a summer wheat, sowing of the progeny was made on April 1. The hybrids produced by direct crosses and back-crosses were absolutely identical as to their morphological characters. Neither showed they any distinction during the growing period, the summer habit being invariably an obviously dominant character.

The seeds of the F_2 were sown on September 1, 1941 in soil, at the territory of the Tomsk station for testing grain crops. By the time of snow-fall, the shoots were in a good condition.

Out of the parental forms only variety Saratov 0329 and Moscow 24-11 were found in spring 1942 to have survived wintering, the ratio of plants that survived being below 50; the rest perished completely. As to the hybrids F_2, these gave a certain per cent of overwintered plants, in all combinations. And it was quite evi-

dent that the plants that survived wintering best were those crosses, in which the variety Saratov 0329 participated as the mother plaint (Table 2).

Table 2

Ratio of Survival among F_2 Hybrids at the Tomsk Station for Variety Tests in Winter 1941—1942

Component crossed to Saratov 0329	Combination where 0329 ♀	Combination where 0329 ♂	Difference
Moscow 24-11	37.9	28.9	9.0
Dutch winter wheat	34.1	29.5	4.5
Turkestan winter wheat	24.3	9.2	15.1
Garnet	31.5	13.3	18.2
Pamir summer wheat	14.9	11.9	3.0
Mean . . .	28.5	18.6	10.0 ± 2.7

Those of the individual F_2 hybrids that survived gave a satisfactory yield, both quantitatively and qualitatively. Seed was collected from all of them separately and sown at a plot of the Tomsk station on August 25, 1942. In 1943 the yield of selected individual progeny was harvested, from which it may be seen that the most successful of the plants are much likely to give raise to quite satisfactory varieties of winter wheat to be cultivated in the sub-tayga of Siberia (Table 3).

The best individual progeny proved that derived from crosses between the steppe variety 0329 and those from Holland and Central Russia. They show a winter-hardiness that is not below that of the steppe variety 0329, whereas their tillering habit is much superior (sowing was done in nests 10 × 10 cm large), and the yield per unit area is 1.5—4 times that of 0329.

Table 3

Best Individual Progeny of Hybrid Wheats Obtained at the Tomsk Station in 1943 (harvest on 2.VIII) as Compared to Standard 0329 of the Saratov Station (sowing done at every 5th plot)

No.	Initial cross	Morphological characteristics of each individual progeny	Ratio of seedlings survived		Number of ears per 1 m²		Yield of grain per 1 m² in g	
			in a given individual progeny	half-sum for nearest standard	in a given individual progeny	half-sum for nearest standard	in a given individual progeny	half-sum for nearest standard
4	Saratov 0329 Moscow 24-11	*Lutescens* without admixture	100	98±3	252	210±17	138	66±17
5	Do	*Lutescens* with admixture of *Erythrospermum*	100	98±3	262	210±17	100	66±17
9	»	Do	95	89±7	304	193±1	110	52±0
14	»	»	93	66±16	291	133±58	163	39±13
53	Saratov 0329 Holland winter wheat	*Lutescens* without admixture	80	62±34	317	122±68	142	45±23
54	Do	Do	80	62±34	310	122±68	128	45±23
55	»	»	71	60±19	420	148±73	225	73±31

Now, if we take into account that, as a rule, the yield of summer wheats at the Tomsk region is some 2—2.5 times that of the best winter varieties available, and that in 1943 the yield of the former proved but 1.7 times above that of the Saratov winter variety 0329, there is full ground to expect that the individual progeny of separate hybrids enumerated in Table 3, if improved by breeding, will give practically valuable varieties of winter wheats that will pay at the sub-tayga zone of Siberia, being better producers than the best summer varieties.

It is premature, of course, to claim at having solved the problem of winter wheat to be cultivated under conditions of the sub-tayga zone of Siberia. But the crossing of steppe winter wheats to winter wheats of zones with an excess of moisture seems to be highly promising in this respect.

In concluding we wish to acknowledge the direct participation in the breeding work described of the Tomsk station for variety-tests of grain crops, represented by the head of the said station, V. A. Korezky.

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PLANT PHYSIOLOGY

ON THE POSSIBILITY OF RUBBER FORMATION BY PLANTS ON HETEROTROPHIC NUTRITION WITH CARBOHYDRATES

By A. A. PROKOFIEV

(Communicated by A. N. Bach, Member of the Academy, 5.VI.1943)

It has been shown by numerous analyses that in such rubber-plants as tau-saghyz, kok-saghyz and krym-saghyz their rubber is largely being accumulated in roots. The stalks are much poorer in rubber, and the leaves still more so. The lateral roots are known to be much richer in rubber as compared to the main root ([1],[2]). Thus, in the chief herbaceous rubber-plants it is the root system which, irrespective of its morphology, accumulates almost the whole rubber contained in the plant. The knowledge of this fact alone does not give us ground, however, to consider the roots as rubber synthesizing organs, for we know many cases where the formation of a product proceeds in one organ, while considerable amounts of it are accumulated in another. Data secured by different writers being at variance, we are unable to decide neither on the place of the synthesis of rubber in plants, nor on the rôle of each separate plant organ in this process ([3–5]).

This was the circumstance that stimulated us to undertake a special study for which, as one of the means to approach the question, the method of sterile cultures was chosen ([6]). As material for our work tau-saghyz was chiefly made use of; the rest of root rubber-plants that yielded essentially the same results, occupied a secondary place.

The aim of the preliminary experiments, carried out both under sterile and unsterile conditions, was to find out how long a plant may subsist in the dark at the expense of the reserve material stored in its seed. Both sterile and unsterile cultures gave similar results. The plants kept in the dark showed elongated internodes and etiolated cotyledons. All kok-saghyz and krym-saghyz plants perished 12—15 days after sprouting, and after 18—21 days all plants of tau-saghyz (ecotypes from Kaynar-bastav and Jelagan-ata).

Thus, a 2—3 weak withdrawal of light proved sufficient fully to consume the reserves stored in the seed. The greater longevity of tau-saghyz in the dark seems to be due to the larger size of its seed and, consequently, to the larger amount of nutrient material in them.

The data obtained in these tests served as a basis for establishing the length of the experiments on the heterotrophic nutrition of tau-saghyz with carbohydrates and other organic substances.

In so far as the growth of the plants and the accumulation of rubber in them might easily be affected by many factors, a series of special investigations had to be done, before we were able to choose optimal conditions, as literature contains no information whatever as to sterile cultures of rubber plants.

a) Growth of Tau-saghyz in the Dark as Affected by the pH of the Medium. Data secured from pot experiments point

to tau-saghyz being greatly susceptible to increased acidity (7). Our tests go fully to confirm these observations; we have found, besides, that when grown in the dark, the susceptibility of this plant to the acidity of the medium is still increased.

The tests were carried out under sterile conditions, Knop's solution serving as a medium. As a source of hydrocarbon use was made of carbohydrates. The necessary value of the pH was attained by varying the relations between acid and alkaline phosphate. The culture was kept in test-tubes, on 1.5 per cent agar. The effect of the pH value upon the growth of tau-saghyz in the dark is brought out in Table 1.

As may be seen from the table, by reducing pH to 6—6.4 conditions are created that impede a normal growth of tau-saghyz, and that of its root system, in particular.

Analogous results were obtained when cultivating isolated tau-saghyz roots under sterile conditions. The culture was kept largely on the same nutrient media as that of the plant as a whole. Young tau-saghyz seedlings, grown under sterile conditions from desinfected seeds in Petri dishes, had their root-tips cut about 5—6 mm long*.

Table 1

Source of hydrocarbon	Period of cultivation in 24h. days	pH of medium	Root length of individual plants				Mean root length in set in mm	State of the plant by the end of experiment
			I	II	III	IV		
Saccharose	50	7.8	24	70	100	115	77	Cotyledons and leaflets dying off; weak growth of roots in progress
»	50	6.1	0*	0*	2	6	2	Roots stopped growing altogether 12 days upon sowing
Glucose	50	7.3	35	60	64	106	66	Cotyledons and leaflets dying off; roots go on growing poorly
»	50	6.0	0*	1	8	15	6	Roots stopped growing as soon as 9 days upon sowing
Fructose	21	6.4	2	12	15	24	13	Experiment suspended after 21 days because of total standstill of root growth and dying-off aboveground mass

* By 0 (zero) seeds are denoted that germinated, but failed to give any appreciable rootlet.

The bits cut-off were transferred under sterile conditions into glasses of a 200—250 cm³ capacity. A good aeration of the roots was secured by the small amount of the nutrient solution (25—30 cm³). The growth of the roots was recorded by measuring their length.

The experiments have shown that in media with a pH below 7 the roots were growing very slowly and stopped growing at all before long. The roots soon assumed a brown colour and died off. When transferred to a medium with a pH=3.8, root-tips of tau-saghyz and tomato plants assumed a brown colour after a few hours; the roots of tau-saghyz died off, while those of tomato plants continued growing and gave numerous lateral rootlets.

On the ground of the experiments described we are brought to conclude that tau-saghyz is highly susceptible to the reaction of the medium, and shows an outspoken preference for media with a pH above 7, i. e. alkaline ones. Acid media

* Attempts to reduce the length of the root-tips to be cultivated to 2—3 mm have led to no good results: the bits developed a poor growth and stopped growing soon.

cause a standstill of the growth of roots, the tissue becomes brown, and next follow dying-off the root and death of the plant as a whole.

Microscopic examination of roots that perished as a result of increased acidity of the medium, has shown that the cells of the parenchyma contain a considerable amount of astringent substances, like the roots of plants that perished on plots. It seems probable that one of the causes responsible for the failure of tau-saghyz plants on plantations is the general or local acidity of the soil.

b) Growth of Tau-saghyz in the Dark as Affected by the Nature of the Hydrocarbon Source. The elimination of photosynthesis when cultivating plants in the dark made it necessary to choose an adequate hydrocarbon source that would be able to compensate to some extent for the lack of autotrophic nutrition.

The cultures were grown under sterile conditions in the dark. As a medium, Knop's solution was used, with addition of a corresponding source of hydrocarbon. The replication was a tenfold one. The initial value of the pH was 7.6, the ultimate (after a month of cultivation), 7.2—7.4.

The following substances were put to test: alcohols (ethyl alcohol, glycerine, mannite), pentoses (arabinose, xylose), hexoses (glucose, fructose), disaccharides (saccharose). The substances here listed were introduced into the media at a ratio of 1.5 and 2 per cent, save for ethyl alcohol that was tested at a concentration of 0.1 and 0.2 per cent, and for glycerine whose concentration was 1 and 1.5 per cent. Sowing was done with germinated seeds. Records were taken one month upon sowing. The results are illustrated by Table 2.

Table 2

Source of hydrocarbon	Mean root length at recording in mm	State of plant by the time of recording	Number of plants with lateral roots (in % to the total number of plants)
Alcohol	9	All plants perished	0
Glycerine . .	15	71 % of plants perished	0
Mannite . . .	18	78 % » » »	0
Arabinose . .	30	100 % » » »	0
Xylose	31	33 % » » »	0
Saccharose . .	41	37 % » » »	25
Glucose . . .	57	All plants survived	43
Fructose . . .	81	» » »	29

As may be seen from this table, the best growth of tau-saghyz in the dark is attained with hexoses which (along with saccharose) were used by us in our subsequent tests with hydrocarbon sources for the heterotrophic nutrition of tau-saghyz plants.

c) Effect of Various Sources of Vitamins upon the Growth of Tau-saghyz Plants and their Severed Roots under Heterotrophic Carbohydrate Nutrition. Assuming that when cultivated in the dark, the poor growth of tau-saghyz plants and especially of their severed roots may possibly be due to lack or deficiency of vitamins, various vitaminous substrata were tested, including extracts from mycelia of various fungi. These tests were based on the observation that the isolated tau-saghyz roots that had casually been infected with the fungus, developed a more intensive growth than those which remained under conditions of perfect sterility.

Testing of extracts from various fungi, both isolated by us from the seed of tau-saghyz and obtained from known strains (*Penicillium* AD_1, *Aspergillus niger*, *Aspergillus flavus*) showed that an addition of insignificant amounts of some of

these extracts (*Penicillium* AD_1, fungus No. 5 isolated by us from seed of tau-saghyz and subjected to no closer determination) actually enhanced the growth of roots and of integer tau-saghyz plants.

Thus, for example, within 7 days of cultivation isolated tau-saghyz roots gave in the control an increment by 15 mm. When added some extract from fungus No. 5, the roots increased in length by 36 mm within the same period of time. When added some fungus extract, the air-dry weight of plants grown in the dark on saccharose equaled 31 mg, without extract it was 20 mg. Especially beneficial is the effect produced upon the growth of the plant by the excretion of a live fungus, separated from the severed tau-saghyz root by a semi-permeable membrane made of cellophane.

d) Growth of Tau-saghyz in the Dark as Affected by the Nature of Nitrogen Supplied. In our attempts to obtain optimal development of our tau-saghyz plants in the dark, we could not overlook the question of nitrogen nutrition of this plant. The various tests on cultivating severed roots and entire tau-saghyz plants upon different nitrogen sources (asparagin, salts of aspartic acid, nitrates) have shown that on organic nitrogen (asparagin) the development of tau-saghyz is not inferior to that on nitrates. The best to enjoy it were the isolated roots of tau-saghyz.

All the data discussed here were taken into account when growing in the dark integer tau-saghyz plants, as well as when cultivating severed roots of the plant upon synthetic media under sterile conditions*.

Both entire plants and isolated roots were subjected to microscopic examination; it was found that under conditions of heterotrophic carbohydrates tau-saghyz plants develop a normal system of laticiferous vessels, as would plants of a corresponding age in the case of autotrophic nutrition.

When analysed, the latex that ran out of injured plants or their isolated rootf was found to contain numerous rubber globuli. The shape of these was typical os young tau-saghyz plants, or of their roots. In severed roots and plants 1—2 -month old the rubber globules are small, spherical in shape. Those of 3-month-old plants are larger and dolioform in shape. Similar age variations in the shape and size of rubber globules are invariably observed in plants grown on plots.

By applying the bromination method ([8]) we were able to show that the formation and accumulation of rubber actually proceeds in laticiferous vessels of the roots of plants grown under conditions of heterotrophic nutrition.

From the data obtained we infer that in root rubber-plants the synthesis of rubber takes place in the latex system. The formation of rubber in the root stands in no direct relation to photosynthesis and may proceed even without any above-ground parts of the plant available, provided there is supply of carbohydrates. The hypothesis suggested by Novikov, Kisselev, Nichiporovich and others ([4], [9], [5]) who believe that the synthesis of rubber takes place in the assimilating tissues of root rubber-plants, should be considered rather doubtful as being in conflict with evidence.

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* When cultivating entire tau-saghyz plants in the dark, we obtained such plants whose dry weight amounted to tens and hundreds of mg. Thus, *e. g.*, at the age of 3 months under the conditions here described some of the plants accumulated up to 143 mg of dry weight. In the first passage isolated roots attained a length of 60—70 mm, the initial length of the root-tip being 5—6 mm.

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PHYSIOLOGY

ON INHERENT AND ACQUIRED FOOD REACTIONS IN REPTILES

By V. V. CHERNOMORDIKOV

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In the course of evolution of the animal kingdom, inherent or instinctive reactions are gradually loosing their importance, while temporary or conditioned connexions (conditioned reflexes, habits) gain an increasingly important rôle.

Thus, if the complex food reaction of solitary predatory wasps is, according to Pecham's observations (3), a hereditary action pattern, the mice hunting in cats is, as shown by Rogers(4), made up of acquired reactions which are formed in kittens at such an early stage, that many writers are erroneously interpreting them as inherent reactions. The reaction of a dog to the aspect and smell of meat is likewise an acquired reaction (5), for puppies reared up to 6 months on milk do not respond to any other food.

The present note contains an analysis of the food reactions in some representatives of the class of reptiles, delimiting the hereditary elements of behaviour from the acquired ones.

According to the literary data available, the whole of the complex food reactions in snakes is due to inherent behaviour, while the rôle of training seems to be reduced to a mere perfectioning of the latter. Thus, for example, Werner (6) writes that young boa constrictors (*Boinae*) assume the behaviour of adult animals directly upon hatching. According to observation by Ozol (2), *Vipera ursini* Bonaparte snatches at flies directly upon drying and moulting as soon as hatched.

In the marine turtle (*Caretta caretta*) the inherent behaviour plays but a secondary part; Hooker (1) has established that these reptiles are not guided in their choice of food by instinct, but previously test the edibility of every object they encounter.

No detailed observations on representatives of the order *Sauria* are available. We have investigated the following representatives of this order: *Eumeces schneideri* Daudin, *Anguis fragilis* L., *Lacerta agilis* L. and *L. viridis* Laur., and *Agama sanguinolenta* Pall.

Observations on *Eumeces schneideri* Daudin. Adult animals obtained: 6 from the Apsheron peninsula (4 of them in 1939 and 2, in 1940); 7 individuals from the station Kara-Bata, in 1941. The observations were started directly upon the receipt of the animals. An obvious reaction to mobile food has been recorded, *viz.* snatching at and devouring of meal-worms.

In our observations on young animals we made use of the two individuals obtained in February 1940 from the Apsheron peninsula, where they had been found buried for wintering. Their age has not been determined, but apparently these were very young animals, for traces of the yolk-sack were still observable on their bellies. Their weight was 3.125 and 2.760 g, respectively.

1st Day of Observation. Both animals approach the meal-worms prepared for them; they touch them with their tongues, but do not eat. They also test with the tongue all objects.

2nd Day of Observation. The animals do not eat the larvae. One of the latter is therefore made an incision at the caudal end, and the point of incision put near the mouth of the animal; it is held so until the animal stretches out its tongue and licks it. After the first touch, the typical reaction sets in: the animal swiftly licks the incised part of the larva and swallows it. Here we have to do with an inherent unconditioned reflectory taste reaction.

3rd Day of Observation. The animals are given incised larvae which are being swallowed much more rapidly, for much less time is now necessary for preliminary licking.

4th Day of Observation and Subsequent Days. The time necessary for the preliminary testing of the larvae gradually decreases.

11th Day of Observation. One of the animals eats intact larvae without licking them: it rapidly approaches the larvae put into the terrarium, snatches them and begins to swallow.

13th Day of Observation. The animals show the reaction typical of adult individuals: they snatch and swallow the larvae creeping about.

Further Observations. The animals run at any moving food, *e. g.*, moth, small beetles.

Observations on *Anguis fragilis* L. We failed to record any reaction to mobile food (meal-worms) in any of the three adult individuals obtained from Odessa. The animals were feeding on raw chopped flesh which they preliminarily tested with their tongues.

For our observations on young animals use was made of 5 individuals born on August 25, 1940 in the terrarium, from a female newly caught in the environs of Moscow. All of them ate raw chopped flesh and incised meal-worms, the latter after having tested with the tongue the place incised. In the subsequent the reaction underwent the same variation as in *Eumeces schneideri*.

Let us cite here our diary.

September 22. There appeared habits connected with the external appearance of the edible object: the animals snatch at the larvae without licking them, running at them from a distance. Only incised larvae are eaten.

October 23. The animals devour whole unincised larvae; the visual reaction prevails over the taste reaction.

November 19. The animals run after creeping larvae, snatching them away from one another; a reaction to moving prey has appeared.

Conclusions. In both *Eumeces schneideri* Daudin and *Anguis fragilis* L. the reaction to the motion of the prey is an acquired one. At the very first period of their life the animals test the edibility of each object with the aid of the taste analyser, touching with the tongue each object they meet on their way.

Observations on *Lacerta agilis* L. and *L. viridis* Laur. The behaviour of the two species is so much alike that we give a common record for both. As material served young animals, artificially hatched from eggs in a thermostat; 9 individuals of *L. agilis* were hatched in May 1941, *L. viridis* were hatched in May 1941 (7 individuals) and in June (6 individuals). The observations were started directly upon hatching.

Below is given a summary of our records.

1st Day of Observation. The animals are very shy, their movements are awkward because of the yolk-sack that is dragging after them; they do not react to the food offered.

2nd Day of Observation. The yolk-sack is resorbed, the behaviour of the animals does not differ from that of adult individuals. They adroitly hunt mobile prey: flying moth, small meal-worms. Every object they run across they test with their tongue to find out whether it is edible or not. Being repeatedly given immobile food, the animals begin to swallow it without preliminary testing with the tongue, which points to that visual reaction has been formed.

Conclusions. The reaction to mobile prey in *Lacerta agilis* L. and *L. viridis* Laur. is an inherent one and does not differ from that in adult animals. Immobile food-objects are distinguished by means of the taste analyser; in the subsequent a habit is developed to the external appearance of the food-object, which is observed in adult animals.

Observations on *Agama sanguinolenta* Pall. We had at our disposal 6 individuals hatched from eggs in a thermostat early in July 1941.

1st Day of Observation. The animals are very shy, their movements are clumpsy because of the yolk-sack they trail behind them; they show no reaction to the food offered.

2nd Day of Observation. The yolk-sack is resorbed, the behaviour of the young is similar to that of adult animals. They are alert in their hunt after mobile food, flying moth, small meal-worms. Immobile food objects they are not acquainted with (eggs of ants, bits of raw flesh, *etc.*) are left without any reaction on their part, as on that of adult animals.

Conclusions. The reaction to mobile prey is inherent in *Agama sanguinolenta*; there is no reaction to immobile objects.

Thus, according to the innate food reaction reptiles may be divided into three groups:

Group 1. The innate food reaction does not differ from that in adult animals (snakes, *Agama sanguinolenta*).

Group 2. The reaction to mobile food is inherent; immobile objects call forth an unconditioned taste reaction which is later transformed into a conditioned visual reaction (*Lacerta agilis* and *L. viridis*). Food reaction of mixed type.

Group 3. Each object is tested for its edibility; for those of them which produce an unconditioned taste reaction, a conditioned visual reaction is worked out (*Eumeces schneideri*, *Anguis fragilis*); acquired food reaction.

The existence of a given group of food reaction may be accounted for by the ecology of feeding of a given species. Thus, for example, the snakes feed, as a rule, on mobile prey, and an inherent reaction to it is undoubtedly a valuable character which enables the animal to get the necessary food from the very first day of its life. The existence in *Lacerta agilis* and *L. viridis* of a mixed type of food reaction creates a lability of the behaviour of the young animals that are able to snatch at both mobile and immobile food objects.

The feeding of young during the first period of their life on poorly mobile prey evoking an unconditioned taste reaction, necessitates an unconditioned reflectory activity only, which is the case with *Eumeces schneideri* and *Anguis fragilis*.

The class of reptiles is thus found to include all types of food reaction, from the hereditary action pattern to temporary conditioned connexions; there are also intermediate forms between both types of reaction, in which case the behaviour of the animal is made up both by inherent and acquired reactions.

The presence of a given type of behaviour should not be considered as a definite stage in the evolution of the higher nervous activity, it should be brought into relation with the ecology of feeding of the species concerned.

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ZOOLOGY

CONTRIBUTION TO THE TAXONOMY OF PALAEARCTIC *MICROTINAE*

By S. I. OGNEV

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While examining and treating vast collections of *Microtinae* for volume VII of the «Mammals of the USSR», I ran across a number of new and highly interesting data that are greatly important for the taxonomy and zoogeography of palaearctic mammals. Some of the most important facts are exposed here briefly.

I. In the first line, let us examine the subspecies of the flatskulled vole, *Platycranius streltzovi* Kastsch.

1) *Platycranius streltzovi streltzovi* Kastsch. (1899).

Type Locality. Central Altay, shore of lake Tenga.

Diagnosis. Zygomatic arches moderately wide (15—17 mm; usually about 16 mm); interorbital region with a slight constriction in its hind part: the interorbital region forms no sharp concavity in the profile of the skull; *processus angularis* of the lower jaw relatively thin. The molars have broad and sharpened salient angles.

The upper parts are of a relatively dark grey, with a browny tinge (hair brown, Ridgway, 1912, pl. XLVI); the tips of the long hairs are of intense brown (fuscous, XLVI) which makes the general colouring of the fur appreciably darker. On the abdominal side there is a well pronounced admixture of pale-yellow to the grey-white background of the fur.

Geographical distribution. Central Altay, Zaysan depression, Tarbagatay.

2) *Platycranius streltzovi desertorum* ex Kastsch. (1901, nomen nudum).

Type Locality. *S.* 19342 ♂ senex, 19.VII 1937, Mt. Kara-Kuus, 70 km south of Karkaralinsk.

I was able to investigate a fine series (106 skins and 10 alcohol preparations) kindly put at my disposal by E. S. Ptushenko and A. P. Rasorenova, and to give a diagnosis of this race, which has once been pointed out, but not described by N. F. Kastschenko (1).

Diagnosis. Zygomatic arches relatively broad, especially in the medium part (16.1—17.7 mm, usually about 17 mm); interorbital space with a very slight constriction in its hind portion; interorbital region forms no sharp concavity on the profile of the skull; *processus angularis* relatively thin. Molars with wide rounded salient angles. Colour of back paler than that in the nominal race. Against the background of the back fur the pale-yellow tinge is conspicuous (drab, pl. XLVI). Young specimens greyer than old ones; the fur on the back of the young is mouse-grey, often darkened by the development of dark-brown tips of the long back-hairs, especially in the lower dorsal region. The pale-yellow tinge is well pronounced on the belly, especially in old specimens; in young the abdominal region is whitish.

Geographical Distribution. Eastern part of the Karaganda province, Karkaralinsk department; southern part of the Pavlodar province, Bayan-Aul department.

3) *Platycranius streltzovi depressus* subsp. nov.

Type Locality. *S.* 20891 ♀ sen. 5.VII. 1937, Chegan-Burgazy, Chuyskaya mountainous steppe *. Under investigation were 52 skins with skulls and 10 flat skins.

Diagnosis. Zygomatic arches moderately broad, like in the nominal race; interorbital region with an observable constriction in its posterior part; the interorbital region forms a sharp concavity on the profile of the skull, the occipital region of the skull being conspicuously elevated behind it; *processus angularis* of the lower jaw relatively thickened. Molar teeth with thin, sharpened salient angles.

The colour of the back is a light-grey (mouse grey, pl. LI), with a scarcely pronounced pale-yellow admixture (drab or light drab, pl. XLVI); abdominal region usually whitish, showing no yellowish tint. In the young individuals the colour of the back is greyer than in old ones; in such animals the abdominal region is a pure white.

Geographical Distribution. Altay, Chuyskaya steppe.

II. New forms of *Microtinae* of the subgenus *Stenocranius* Kastsch.

1) *Microtus* (*Stenocranius*) *gregalis tarbagataicus* subsp. nov.

Type Locality. *S.* 23279 ♂ sen., 8. VII. 1929, Semipalatinsk region, Urdjar province, Podgornoe (Sazy), Tarbagatay mountains **. Aside from the type, 18 skins and skulls were investigated.

Diagnosis. Skull heavier than in the nominal race described by Pallas after specimens from Chulym, and also than in the *Stenocranius gregalis eversmanni* Poljakov from Altay (Uimon). The width of zygomatic arches in fully mature and old specimens about 14 mm and even somewhat more; width of the occipital part of the skull about 11 mm; interorbital constriction seldom attaining 3 mm, usually below 3 mm.

Colour of back clearly distinct from that in related subspecies. On the back the hairs are red-brown-grey, with a well pronounced rusty tinge due to the presence of rather intensely coloured sub-terminal annulations of long hairs; these annulations are of a rusty hue (approaching pinkish-buff or even cinnamon-buff, pl. XXIX). Especially conspicuous are these rusty tinges on the sides of the body. The fur of the abdominal region is a dirty-white, sometimes with a slight yellow tinge. The tail is rusty from above, with a slight grey tinge, and whitish from below. The length of the body attains 130 mm; the length of the tail is up 41 mm (usually about 30 mm or somewhat longer); the greatest length of the skull in fully mature individuals is less than 27 mm (usually 25—26 mm).

Geographical Distribution. Central Tarbagatay, near vill. Podgornoe and at the Narym mountain range, in Alpine meadows, near the source of the river Sarymsak.

2) *Microtus* (*Stenocranius*) *gregalis tundrae* subsp. nov.

Type Locality. *S.* 25457 ♀ sen., 25. VII. 1935, near the station Tundra, 40 km south of the town Arkhangelsk ***. Aside from the type, two ecotypes have been investigated.

Diagnosis. This vole most closely approaching *Microtus* (*Stenocranius*) *gregalis major* Ogn., but differing from this subspecies by its smaller and more delicate skull. The greatest length of the latter in fully mature specimens attains 26—26.7 mm, the zygomatic arches are massive and attain 15 mm. Interorbital space 3 mm wide, the width of the occipital part of the skull 11—11.9 mm. The colour of the upper parts is appreciably paler than in *M.* (*S*) *gregalis major* Ogn. The fur of the back is a yellow-grey, with a yellowish tinge, and in some

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* Material collected by A. M. Kolossov. Collection of the Moscow State University.
** B. A. Kuznetzov. Collection of the Moscow State University.
*** V. J. Parovshchikov. Collection of the Moscow State University.

individuals, with a faint rusty hue. On the sides of the body the fur is paler and more yellow than on the back. On the sides the coloration gradually passes into the dirty-white of the belly whose fur shows occasionally a yellowish tint. Tail very short, clearly bicoloured: pale-yellow, rusty-grey from above, and pure white from below.

Body length 120—124 mm; foot 17.4—17.6 mm; tail 19 mm. Body weight 21—31 g.

Taxonomic Notes. A well-distinct geographical form, resembling by its outward appearance *S. g. major* Ogn., from which it can at once be distinguished after its very short, sharply bicoloured tail, a paler colour of the back and a relatively shorter skull. Among all the subgenus *Stenocranius* it is the only subspecies whose geographical distribution extends so far to the West.

III. New subspecies of Northern grass mouse *Microtus agrestis argyropoli* subsp. nov.

Type Locality. *S.* 25289 ♀ sen., 12.VII. 1938; South Urals, valley of Inzer river. Besides the type, 17 specimens were examined.

Diagnosis. Easily and distinctly separated from the nominal Swedish form, as well as from *Microtus agrestis* from the Northern region of this country after its relatively dull, greyish colour. The general background of the back hair is a pale, dull greyish-yellow, with a slight rusty tint. The general colour of the fur is intermediate between hair brown, drab, pl. XLVI, Rigdway, 1913, *saccardos umber* with a tinge of snuff-brown, pl. XXIX. The colour of the back hair in this race approaches that in *M. agrestis mongol*, but is still duller and greyer, with a slighter rusty hue. On the sides of the body the colour becomes lighter and yellower. The hairs of the abdominal region are a light drab in most individuals. The tail is bicoloured: brown-grey from above, and whitish from below. The legs are covered in their upper part with grey-yellow-whitish hairs. In young the fur colour is more of a grey, than in adult. In a vast majority of these *Microtinae* the posterior-interior angle of M^1 bears no additional posterior salient angles. The salient angles on the molars are not wide (when measured across), this character resembling the structure of the respective teeth in the Siberian race (*M. a. mongol* Thos.). The enamel loop at the hind end of M^3 is bent and well-developed.

The form is a relatively large one. Length of head and body (in fully mature specimens) 107—128 mm (usually about 116 mm); tail length 30—42 mm(usually about 35 mm); total length of skull 27—28 mm; condylo-basal length 27—28.8 mm; width of zygomatic bones 14.8—15.7 mm; occipital length (from the hind edge of the interorbital constriction to the most protruding region of the occipital bone) 14.9—15.3 mm; length of the upper tooth row 6.2—6.3 mm.

Taxonomic Notes. As pointed out above, *M. a. argyropoli* subsp. nov. can easily be distinguished from *M. agrestis* of North Russia by its paler coloration. The Northern grass mice from the Northern region show a more intense grey-brown colour of their back summer fur with a warm brown tinge (warm sepia); on the sides the fur is somewhat more browny (snuff-brown); the belly is of a dirty grey-white colour. Besides these peculiarities in their coloration, *M. agrestis* of North Russia is distinguished from *M. a. argyropoli* subsp. nov. by its somewhat longer tail (in the northern specimens the length of the tail* is 28—52.3 mm; mean, 40 mm).

The South-Uralian race is readily distinguished also from the Central Russian by its rather greyish colour of the back fur, and by larger body and skull dimensions. By the colour of the summer fur on the dorsal part of the body the Central Russian form does but slightly differ from the Northern race. When examined in series, it may be noticed that in representatives of the Central Russian race the back fur is somewhat more brown and lighter than in their Northern relatives. The body length in the Central Russian *M. agrestis* is 89—110 mm; length of tail (without the tip hair) 30—38 mm; length of hind foot (without claws) 17—19.5 mm; total

* Without the tip-hairs.

length of skull 25.6—27.5 mm; condylo-basal length of skull 25.3—27.4 mm; width of zygomatic bones 13.7—15.8 mm; occipital length 14—15.2 mm; length of upper tooth row 6.1—6.7 mm.

The *M. agrestis* from Central Russia has been pointed out as a separate race by W. A. Melander who gave it the name of *M. a. pallidus* [type from the former Elninsky department of Smolensk province, near vill. Miloje (2)]. It is very closely related to the form from Central Russia; perhaps, with it may be identified *Microtus agrestis estiae* (3), described after 16 individuals from the Vilsand and Abruka isles. The characters which Reinwaldt (3) takes as a basis for separating this race, are but little typical (*e. g.* a relatively slighter downward inclination of the nasal bones, than in the northern race). This character seems to be dependent on age. The dimensions of *M. a. estiae* Reinw. are comparatively small and fit well within those of *M. a. pallidus* Meland. To judge by the description and coloration, *M. a. estiae* is close to the subspecies from Central Russia. Upon direct comparison they are very likely to prove identical. In this case, by the law of priority, the *M. agrestis* from Central Russia must be left the name given to it by Reinwaldt.

The Northern grass mouse from the South Urals was given by me its name in memory of the gifted Russian zoologist A. I. Argyropulo, who died recently. Argyropulo was the first to notice some typical traits of this form in a series of individuals obtained by him from a locality situated 100 km south of the town Zlatoust(4).

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